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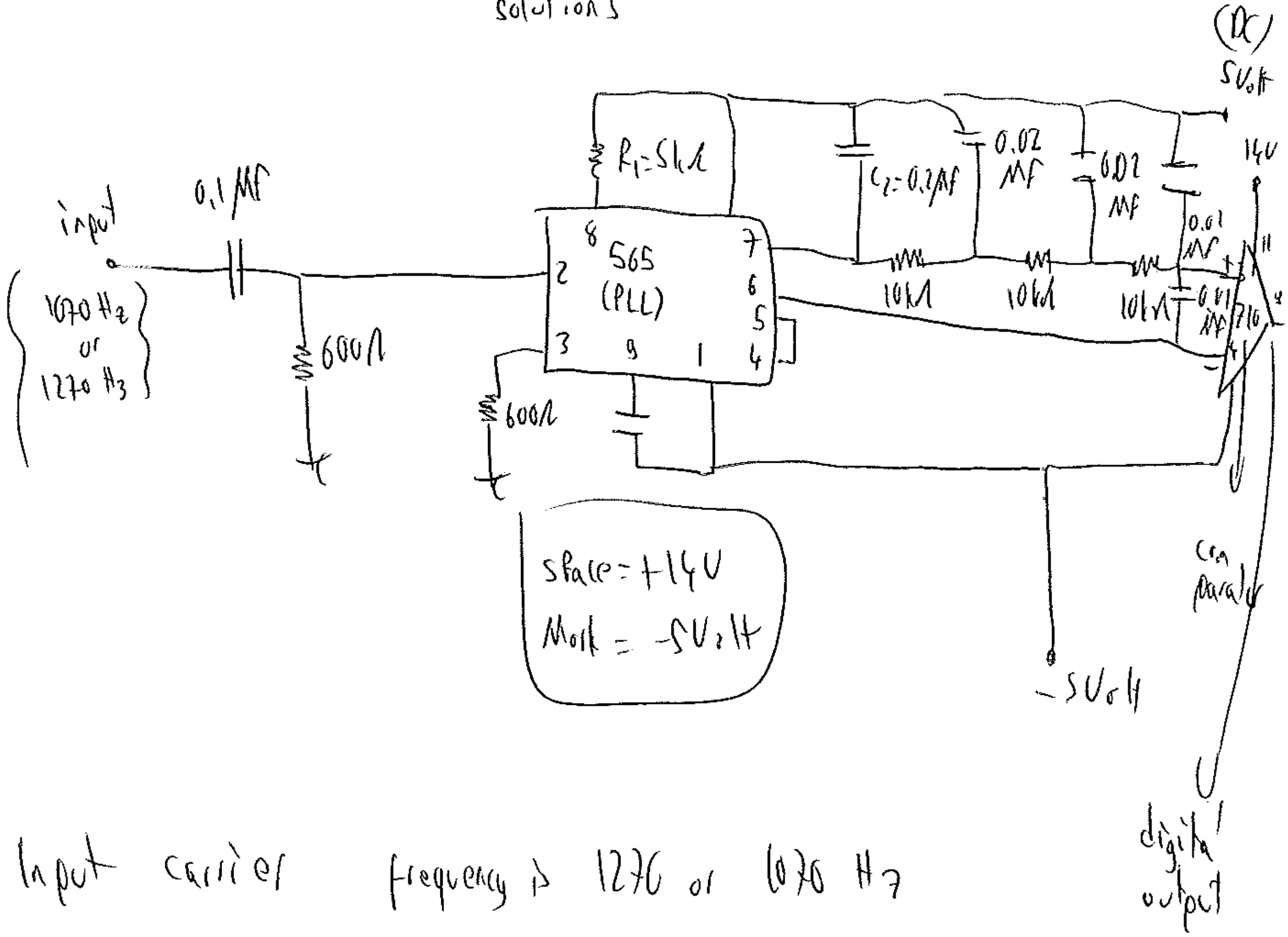
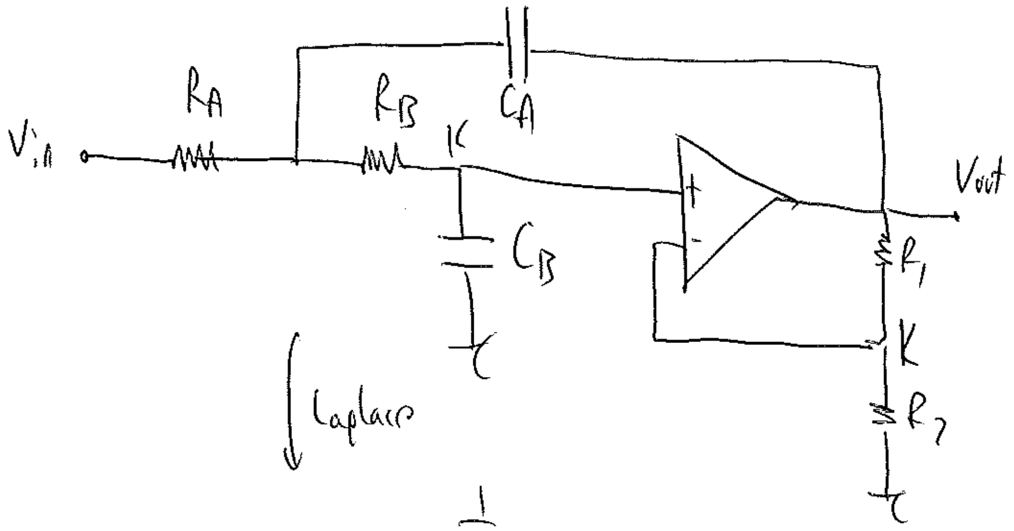
MECE 347 Make-up for final exam solutions

Q1

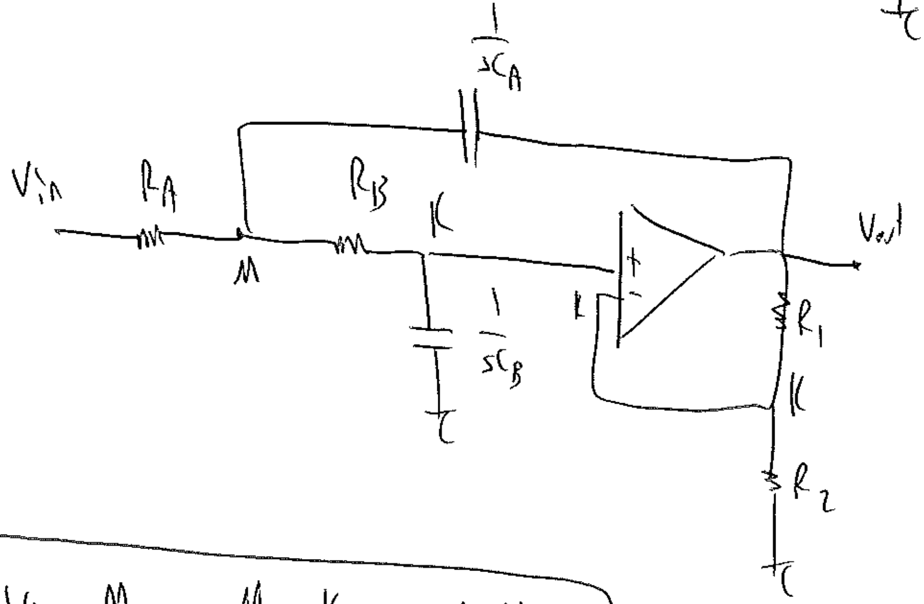


- Input carrier frequency is 1270 or 1070 Hz
- Loop locks to the input frequency and tracks it between two possible frequencies with a corresponding DC shift at the output
- RC ladder filter (three sections of C = 0.02 μF and R = 10kΩ) removes the sum-frequency component
- Free-running frequency adjusted with R_1 so that the DC voltage level at pin 7 (output) of PLL 565 is the same as that of at pin 6.
- Then an input at frequency 1070 Hz will drive the decoder output voltage to a more positive voltage level, driving the digital output to the high level (space or 14 Volt)
- An input at 1270 Hz will correspondingly drive 565 DC output less positive with the digital output, which then drops to the low level -5Volt.

(Q2) The Sallen-Key Low-Pass filter



Laplace



$$\frac{K}{V_{out}} = \frac{R_2}{R_1 + R_2} = X$$

$$K = X V_{out}$$

$$\boxed{\frac{V_{in} - M}{R_A} = \frac{M - K}{R_B} + \frac{M - V_{out}}{\frac{1}{sC_A}}}$$

$$\frac{M}{R_B + \frac{1}{sC_B}} = \frac{K}{\frac{1}{sC_B}}$$

$$\frac{M sC_B}{sR_BC_B + 1} = \frac{K sC_B}{1}$$

$$\boxed{\frac{M}{sR_BC_B + 1} = K}$$

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$$\frac{V_{in}}{R_A} = M\left(\frac{1}{R_A} + \frac{1}{R_B} + sC_A\right] - \frac{K}{R_B} - \frac{V_{out}\, sC_A}{1}$$

$$\frac{V_{in}}{R_A} = \cancel{K}\left[\frac{1}{R_A} + \frac{1}{R_B} + sC_A\right]\left[sR_BC_B + 1\right] - \frac{K}{R_B} - \frac{V_{out}\, sC_A}{1}$$

$$\frac{V_{in}}{R_A} = K\left[\frac{1}{R_A} + \frac{1}{R_B} + sC_A - \frac{1}{R_B} + s^2C_AR_BC_B + sC_B + s\frac{R_B}{R_A}C_B\right] - \frac{V_{out}\, sC_A}{1}$$

$$\frac{V_{in}}{R_A} = X V_{out}\left[\frac{1}{R_A} + s\left(C_A + C_B + \frac{R_B}{R_A}C_B\right) + s^2C_AR_BC_B\right] - \frac{V_{out}\, sC_A}{1}$$

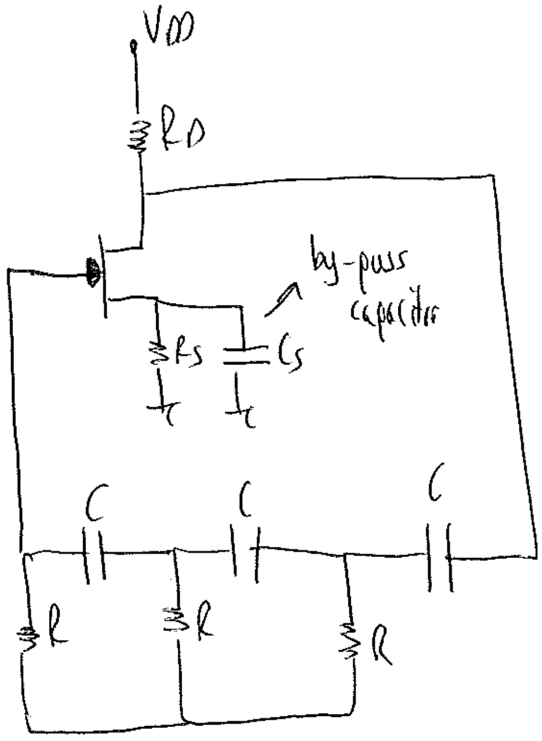
$$\frac{V_{in}}{R_A} = \left(\frac{X}{R_A} + \left[X\left(C_A + C_B + \frac{R_B}{R_A}C_B\right) - C_A\right]s + s^2XC_AC_BR_B\right)$$

$$V_{in} = \left(X + s\left[X\left(C_AR_A + C_BR_A + C_BR_B\right) - C_AR_A\right] + s^2XC_AC_BR_AR_B\right)V_{out}$$

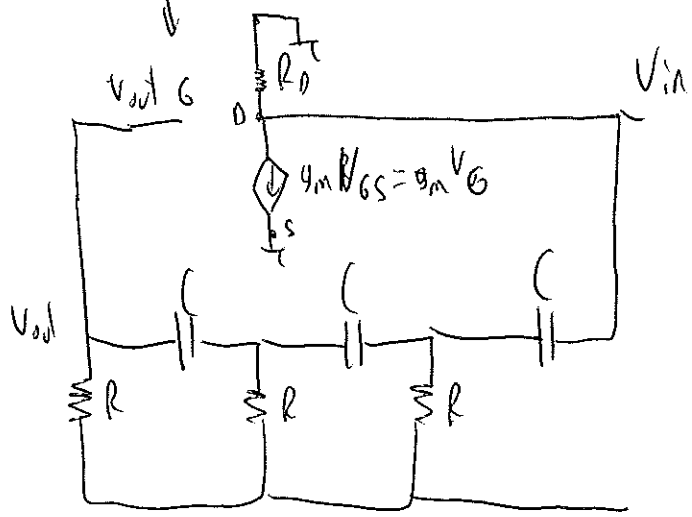
$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2XC_AC_BR_AR_B + s\left[X(C_AR_A + C_BR_A + C_BR_B) - C_AR_A\right] + X}$$

$$\frac{V_{out}}{V_{in}} = \frac{\dfrac{1}{XC_AC_BR_AR_B}}{s^2 + s\left(\dfrac{1}{C_BR_B} + \dfrac{1}{C_AR_B} + \dfrac{1}{C_AR_A} - \dfrac{1}{XC_BR_B}\right) + \dfrac{1}{C_AC_BR_AR_B}}$$

Q3



AC circuit



$V_{in} = V_D$

$V_{out} = V_G$

$V_D = [-g_m V_G] R_D$

$V_D = -g_m R_D V_G$

$V_{in} = -g_m R_D V_{out}$

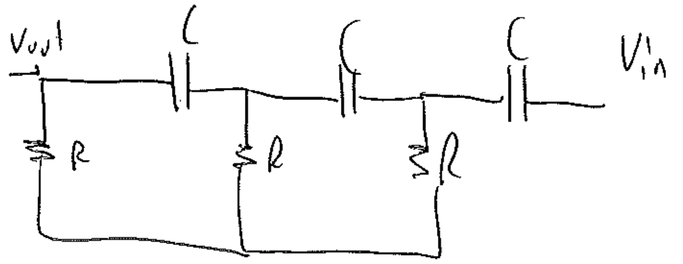
$V_{in} = A V_{out}$

$A = -g_m R_D$

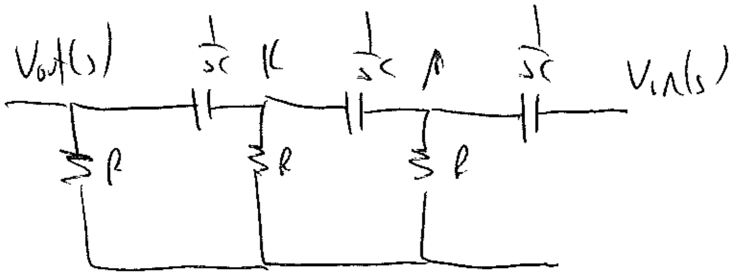
$f = \dfrac{1}{2\pi RC\sqrt{6}}$ → frequency that results with undamped oscillations

Barkhausen criteria

$\dfrac{V_{out}}{V_{in}} \times A = 1$



↓ laplace



$$\frac{V_{out}}{K} = \frac{R}{R+\frac{1}{sC}}$$

$$\boxed{\frac{V_{out}}{K} = \frac{sCR}{sCR+1}}$$

$$K = \frac{sCR+1}{sCR} V_{out}$$

$$\frac{M-K}{\frac{1}{sC}} = \frac{K}{R} + \frac{K-V_{out}}{\frac{1}{sC}}$$

$$sCM = \left[sC + \frac{1}{R}\right]K + KsC - sCV_{out}$$

$$sCM = \left[2sC + \frac{1}{R}\right]K + \frac{-sCV_{out}}{1}$$

$$\cancel{sCM = \left[2sC + \frac{1}{R}\right]\frac{sCR+1}{sC}V_{out} - sRCV_{out}}$$

$$\cancel{s^2C^2M = \left[2s^2C^2R + \frac{sC}{R}\right]\left(sCR+1\right)V_{out} - s^2RC^2V_{out}}$$

$$\cancel{sCM = \left[2sC + \frac{1}{R}\right]\frac{sCR+1}{sC}V_{out} - sCV_{out}}$$

$$\cancel{s^2C^2M = \left[2sC + \frac{1}{R}\right]\left[sCR+1\right]V_{out} - s^2C^2V_{out}}$$

$$\cancel{s^2C^2RM = \left[2sCR+1\right]\left[sCR+1\right]V_{out} - s^2C^2RV_{out}}$$

$$\cancel{s^2C^2RM = \left[2s^2C^2R + 3sCR + 1 - s^2C^2R\right]V_{out}}$$

$$\frac{M}{V_{out}} = \frac{\left[s^2C^2R + 3sCR + 1\right]}{s^2C^2R}$$

$$\cancel{\frac{V_{out}}{V_{in}} = \frac{s^3C^2R}{s^3C^2R+3sCR+1}}$$

$$\frac{V_{in} - M}{\frac{1}{sC}} = \frac{M}{R} + \frac{M - K}{\frac{1}{sC}}$$

$$sCV_{in} - sCM = \frac{M}{R} + sCM - sCK$$

$$sCV_{in} = \left(2sC + \frac{1}{R}\right)M - sCK$$

$$sCRV_{in} = [2sCR+1]M - sCRK$$

$$sCRV_{in} = [2sCR+1]M - sCR\frac{[sCR+1]}{sCR}V_{out}$$

$$sCM = \left[2sC + \frac{1}{R}\right]K + \frac{-sCV_{out}}{1}$$

$$\boxed{sCRM = [2sCR+1]K - sCRV_{out}}$$

$$sCRM = [2sCR+1]\frac{[sCR+1]}{sCR}V_{out} - sCRV_{out}$$

$$s^2C^2R^2M = [2sCR+1][sCR+1]V_{out} - s^2C^2R^2V_{out}$$

$$\boxed{s^2C^2R^2M = [s^2C^2R^2 + 3sCR+1]V_{out}}$$

$$M = \left(\frac{s^2C^2R^2+3sCR+1}{s^2C^2R^2}\right)V_{out}$$

$$sCRV_{in} = [2sCR+1]M - [sCR+1]V_{out}$$

$$s^3C^3R^3V_{in} = [2sCR+1]\left[\frac{s^2C^2R^2+3sCR+1}{s^2C^2R^2}\right]V_{out} - [sCR+1]V_{out}$$

$$s^3C^3R^3V_{in} = [2sCR+1][s^2C^2R^2+3sCR+1]V_{out} - s^2C^2R^2[sCR+1]V_{out}$$

$$s^3C^3R^3V_{in} = \left[2s^3C^3R^3 + 7s^2C^2R^2 + 5sCR + 1 - s^3C^3R^3 - s^2C^2R^2\right]V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{s^3C^3R^3}{s^3C^3R^3 + 6s^2C^2R^2 + 5sCR + 1}$$

$$\angle\frac{V_{out}(s)}{V_{in}(s)}\bigg|_{s=j\omega} = 180^\circ \quad \text{(for oscillation)}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j^3\omega^3C^3R^3}{j^3\omega^3C^3R^3 + 6j^2\omega^2C^2R^2 + 5j\omega CR + 1}$$

only possible when $1-6\omega^2C^2R^2=0$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{-j\omega^3C^3R^3}{-j\omega^3C^3R^3+5j\omega CR+\left[1-6\omega^2C^2R^2\right]}$$

if $1-6\omega^2C^2R^2=0$ $\quad \omega^2=\dfrac{1}{6C^2R^2}$ $\quad \omega=\dfrac{1}{\sqrt{6}\,CR}$

$$f=\frac{1}{2\pi CR\sqrt{6}}$$

Hence if $f=\dfrac{1}{2\pi CR\sqrt{6}} \rightarrow f_{oscillation}$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{-j\omega^3C^3R^3}{-j\omega^3C^3R^3+5j\omega CR} = \frac{-\omega^3C^3R^3}{-\omega^3C^3R^3+5\omega CR}$$

$$\left.\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right|_{\omega=\frac{1}{\sqrt{6}CR}} = \frac{\omega^2C^2R^2}{\omega^2C^2R^2-5}$$

put $\omega=\dfrac{1}{\sqrt{6}\,CR}$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{\frac{1}{6C^2R^2}C^2R^2}{\frac{1}{6C^2R^2}C^2R^2-5} = \frac{\frac{1}{6}}{\frac{1}{6}-5} = \frac{1}{-29}$$

Hence $\left|\dfrac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \dfrac{1}{29}$

Hence $\left|\dfrac{V_{out}}{V_{in}}\right| \times A = 1$ $\qquad$ hence $A=29=\left|-g_mR_D\right|$