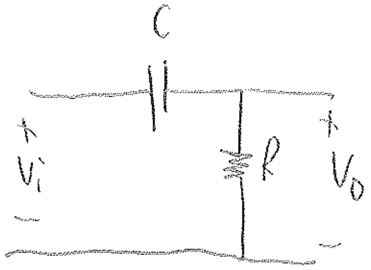
RC circuit (network) Low frequency analysis - bode plot



impedance of $C \to X_C = \frac{1}{j2\pi f C} = \frac{-j}{2\pi f C}$

if $f \to 0$ $|X_C| = \infty\,\Omega$

if $f \to \infty$ $|X_C| = 0\,\Omega$

if frequency is low $|X_C| \to \infty$ [open circuit]

if frequency is high $|X_C| \to 0$ (short circuit)

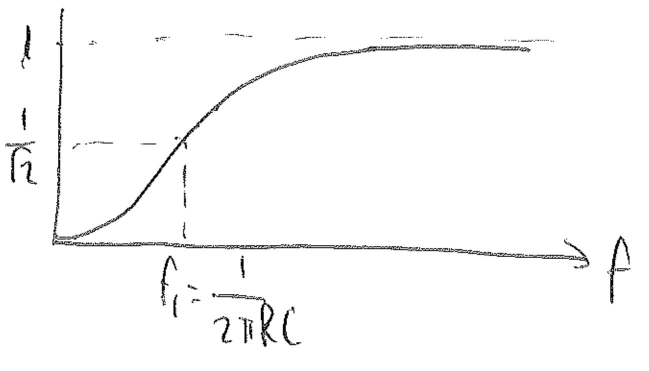
This is a high pass filter

$\frac{V_o}{V_i} = A_V \to$ gain $\quad \frac{V_o}{V_i} = \frac{R}{R + X_C} \quad \left|\frac{V_o}{V_i}\right| = \frac{R}{\sqrt{R^2 + X_C^2}}$

if $X_C = R \Rightarrow \left|\frac{V_o}{V_i}\right| = |A_V| = \frac{1}{\sqrt{2}} = 0.707$

$X_C = \frac{1}{2\pi f_1 C} = R \quad f_1 = \frac{1}{2\pi RC}$ (corner frequency) (half power frequency)

$A_V = \left|\frac{V_o}{V_i}\right|$



Low frequency response of RC circuit

$A_V = \frac{V_o}{V_i} = \frac{R}{R - \frac{j}{2\pi f C}} = \frac{1}{1 - j\left(\frac{f_1}{f}\right)}$

magnitude

$|A_V| = \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}$

$\cancel{\angle A_v = \tan^{-1}\left(\frac{f_1}{f}\right)}$

phase

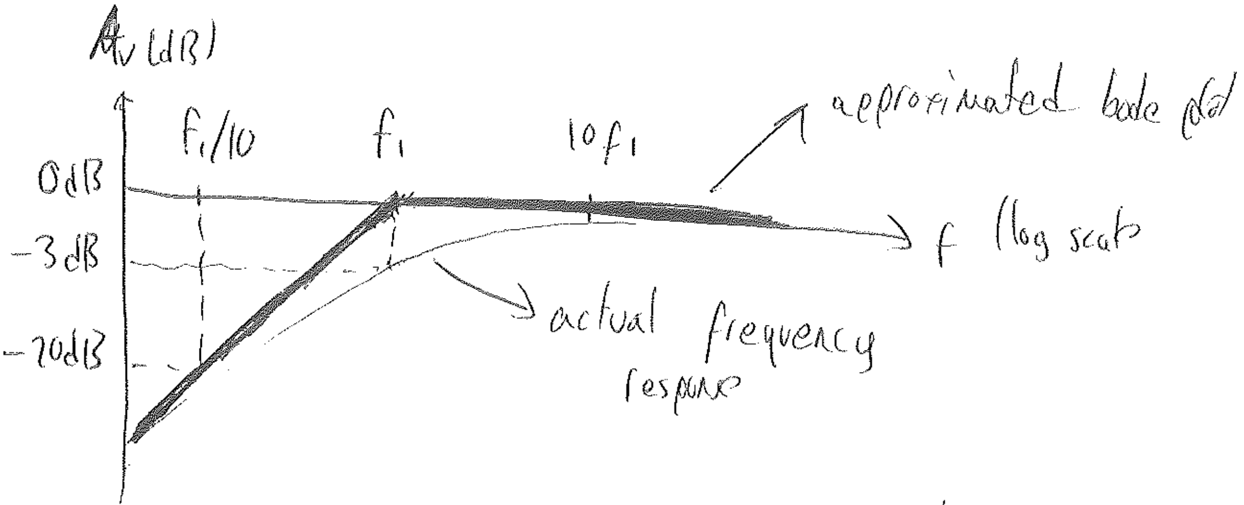
$\theta = \tan^{-1}\left(\frac{f_1}{f}\right) = \angle A_v = \angle \frac{V_o}{V_i} = \angle V_o - \angle V_i$

phase

$$A_{v_{dB}} = 20 \log_{10} \frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$ (The gain in dB)

if $f < f_1$ $\quad A_{v_{dB}} \cong -10\log_{10}\left(\frac{f_1}{f}\right)^2 = -20\log\frac{f_1}{f}$

if $f > f_1$ $\quad A_{v_{dB}} \cong 0$



put $f = f_1$ $\quad A_{v(dB)}\Big|_{f=f_1} = 20\log_{10}\frac{1}{\sqrt{1+\left(\frac{f_1}{f_1}\right)^2}} \cong -3dB$

phase $= \theta = \tan^{-1}\frac{f_1}{f}$

if $f \ll f_1$ $\quad \theta \to 90^\circ$

if $f = f_1$ $\quad \theta = 45^\circ$

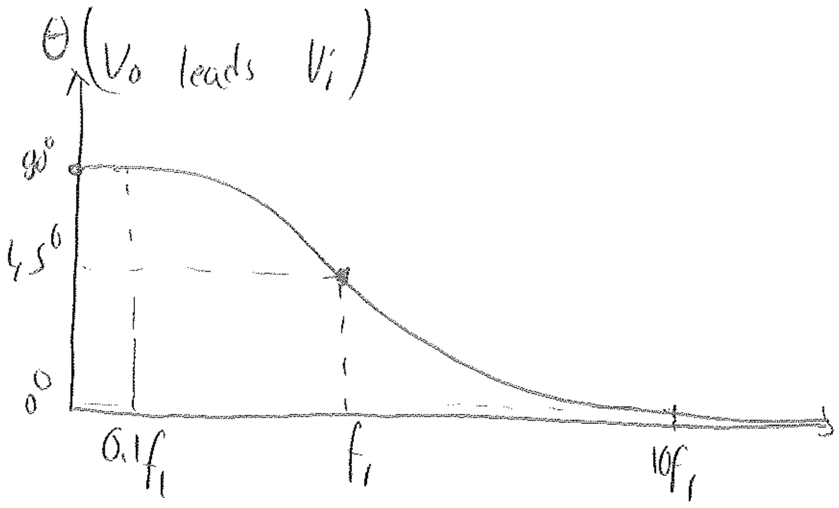
if $f \gg f_1$ $\quad \theta \to 0^\circ$

$\angle V_o - \angle V_i = \angle A_v$

if positive V_o leads V_i

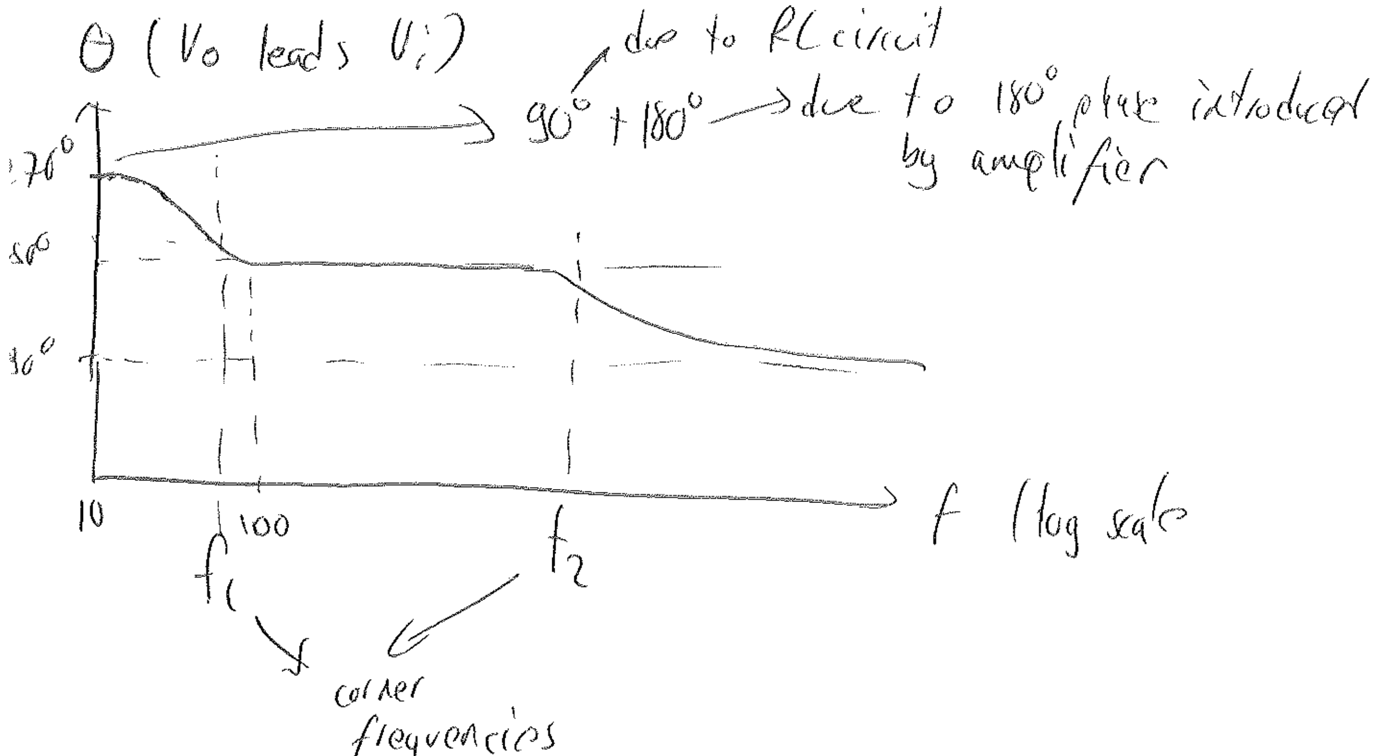
if negative V_o lags V_i

Phase response of RC circuit

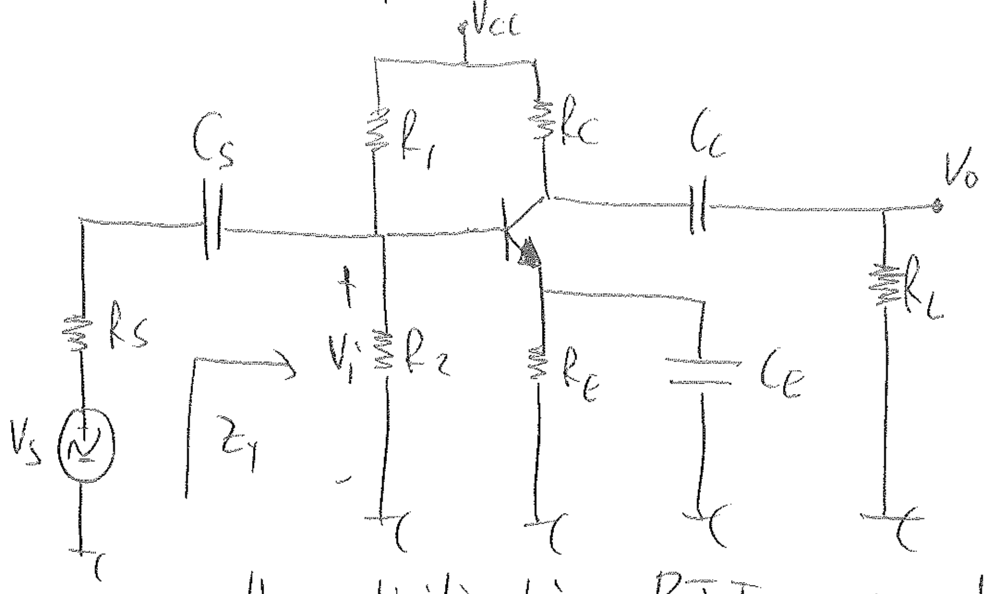


If we add RC coupled network with an amplifier which also introduces a phase shift of 180° we will observe

RC coupled amplifier phase diagram



Low frequency response (BJT) amplifier



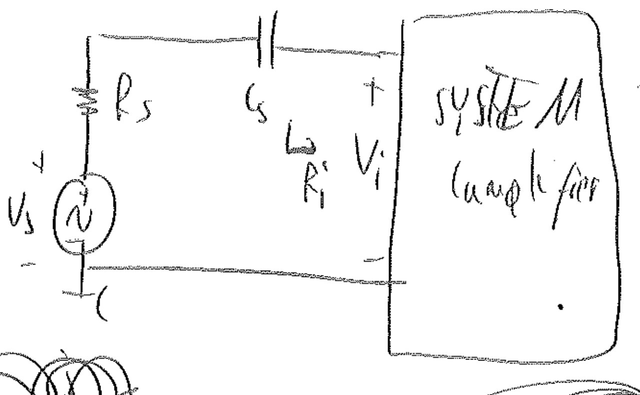
voltage divider bias BJT configuration

$C_s \to$ source coupling capacitance

$C_E, C_C \to$ Collector and Emitter by-pass capacitors

$C_s, C_C, C_E \to$ determines the low frequency response

C_s : normally connected between the applied source and the active device (amplifier)



$\to$ Circuit to determine effect of C_s on low-frequency response

$R_i \to$ input resistance of system (amplifier)

$$X_C = \frac{1}{2\pi f C}$$

$$V_i = \frac{R_i V_s}{R_s + R_i - jX_C}$$

$-jX_C \to$ impedance of capacitor

$$\frac{V_i}{V_s} = \frac{R_i}{R_s + R_i - jX_{C_s}} = \frac{1}{1 + \frac{R_s}{R_i} - j\frac{X_{C_s}}{R_i}} = \frac{1}{\left(1+\frac{R_s}{R_i}\right)\left(1 - j\frac{X_{C_s}}{R_i+R_s}\right)}$$

$$\frac{X_{C_s}}{R_i+R_s} = \frac{1}{2\pi f C_s (R_i+R_s)} \qquad f_i = \frac{1}{2\pi (R_i+R_s) C_s} \rightarrow \text{define}$$

hence

$$\frac{V_i}{V_s} = \frac{1}{\left(1+\frac{R_s}{R_i}\right)\left(1 - j\frac{1}{2\pi f C_s (R_i+R_s)}\right)}$$

$$= \frac{1}{\left(1+\frac{R_s}{R_i}\right)\left(1 - j\frac{f_i}{f}\right)}$$

$$A_v = \frac{V_i}{V_s} = \frac{R_i}{R_i+R_s}\left[\frac{1}{1-j\left(\frac{f_i}{f}\right)}\right]$$

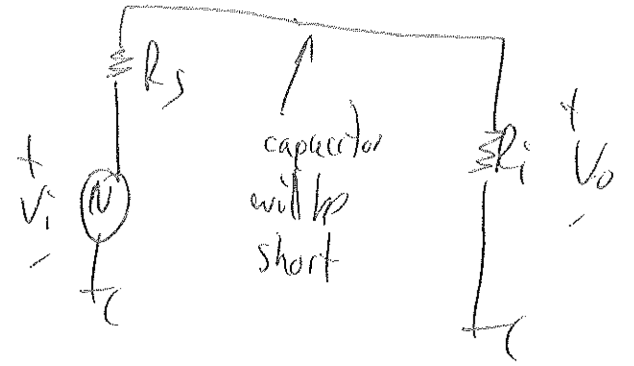
at midband the gain is $A_{v\,mid} = \frac{V_o}{V_i} = \frac{R_i}{R_i+R_s}$

and $$\frac{A_v}{A_{v\,mid}} = \frac{1}{1-j\left(\frac{f_i}{f}\right)}$$

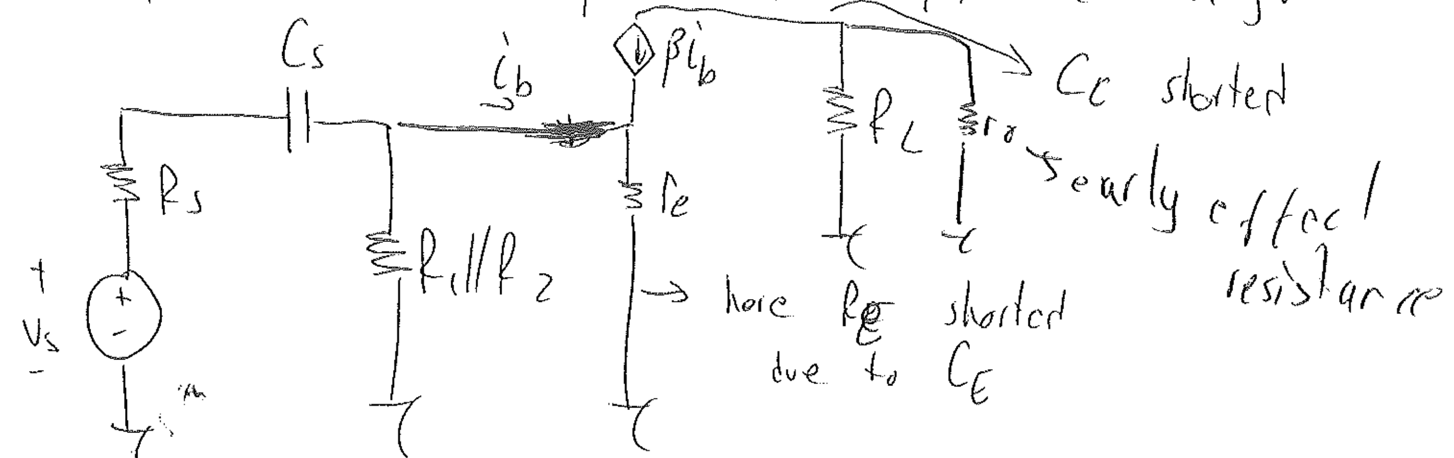
cut-off frequency $f_L = f_i = \frac{1}{2\pi (R_i+R_s) C_s}$

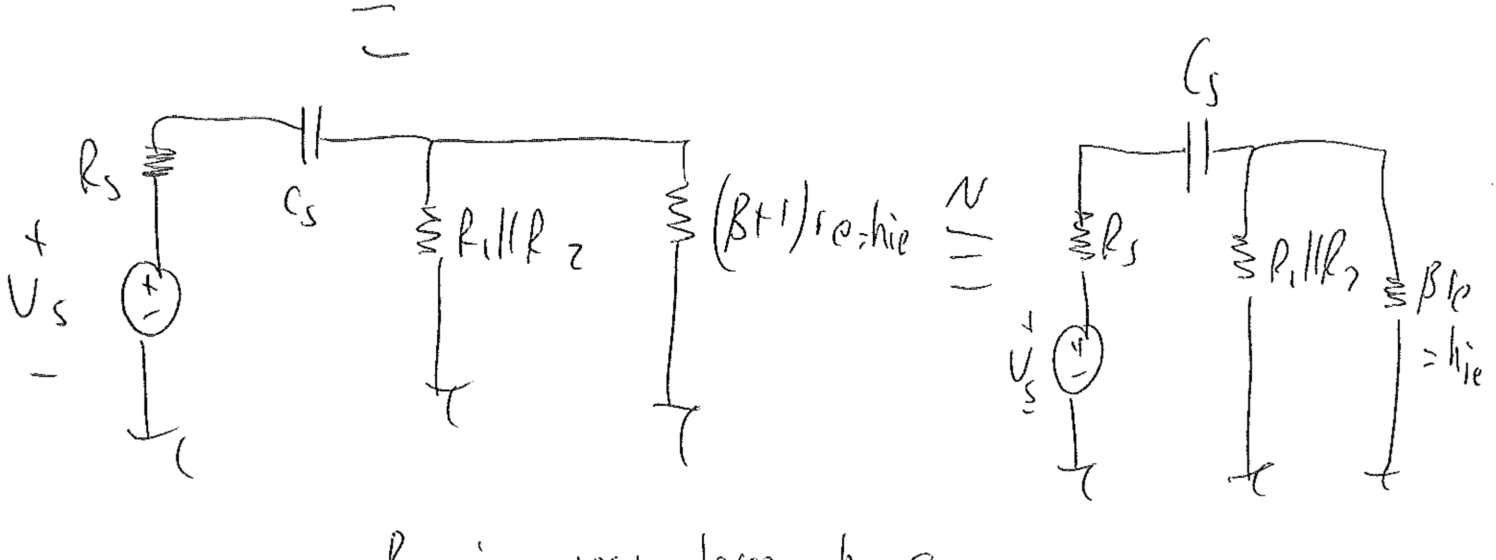
at cut-off frequency V_o will be $\frac{1}{\sqrt{2}}$ of midband gain value $A_{v\,mid}$

at high frequency (High frequency equivalent circuit)



While analysing the effects of C_S we assume C_E and C_C behave perfectly short circuit (since they are by-pass capacitors) using this hypothesis and putting ac equivalent network for transistor (npn) we nearly obtain





β is very large hence

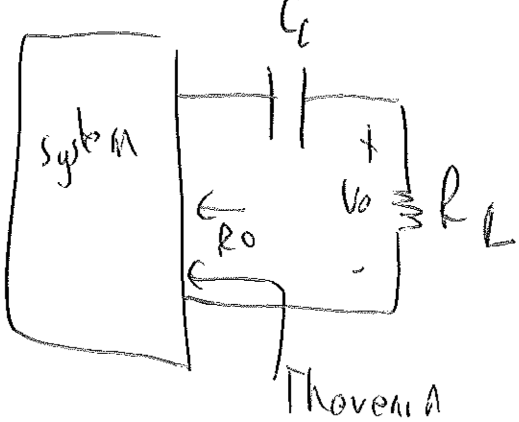
$\beta r_e \cong (\beta+1) r_e$ //

A The value of R_i for $f_{L_s} = \frac{1}{2\pi(R_s+R_i)C_s}$

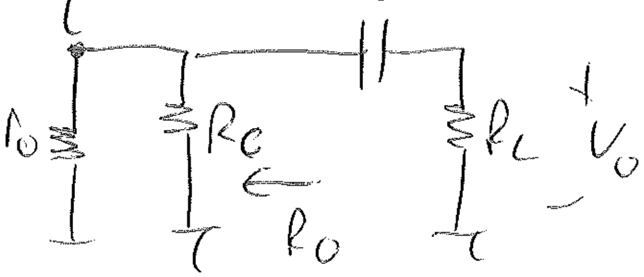
is $R_i = R_1 \| R_2 \| (\beta+1)r_e \cong R_1 \| R_2 \| \beta r_e$

C_c → collector coupling capacitor (connected between output of the active device and applied load)

hence f_{L_c} (low cut-off frequency due to C_c) has following circuit characteristics



if $V_i = 0$ volt + AC equivalent circuit



$$\boxed{f_{L_C} = \frac{1}{2\pi(R_o+R_L)C_c}}$$ → low cut off frequency due to C_c

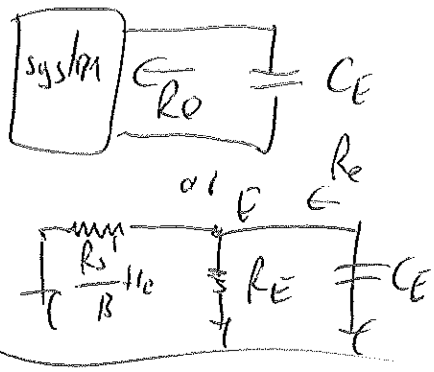
$$\boxed{R_o = R_C \| r_o}$$

To determine f_{LE} → The cut of frequency due to C_E

$$f_{LE} = \frac{1}{2\pi R_e C_E}$$

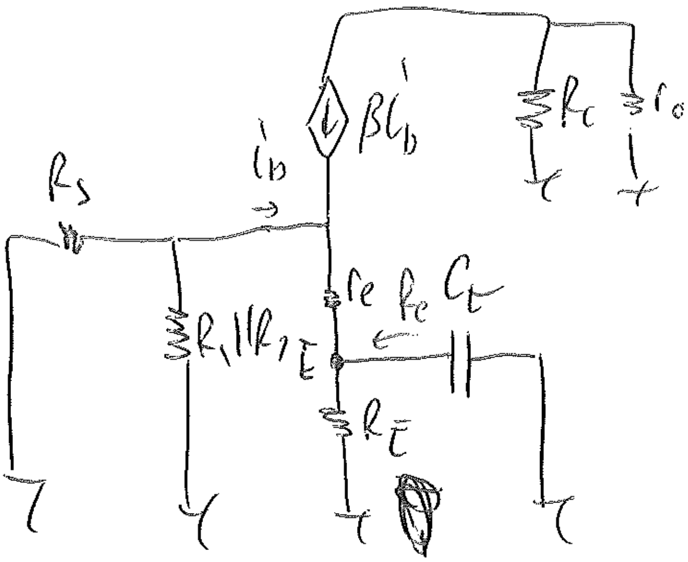
↓ cut-of frequency due to C_E

to determine ~~effect of~~ C_E ~~with~~ R_e we have the circuit below



$R_s' = R_s \| R_1 \| R_2$

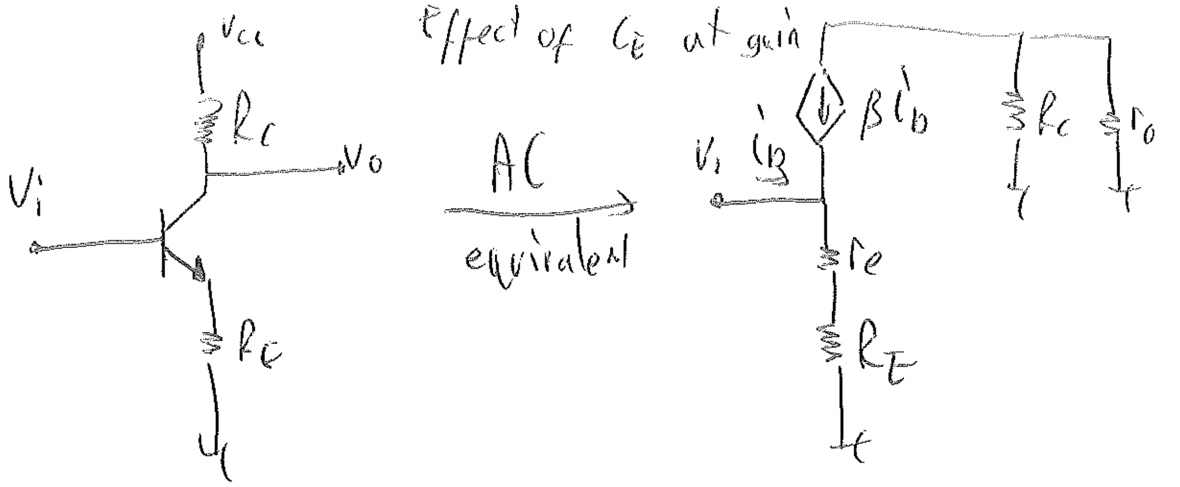
The circuit to obtain R_o



=

$$R_o = R_E \,\Big\|\, \left[\frac{R_s'}{(\beta+1)} + r_e\right] = R_E \,\Big\|\, \left(\frac{R_s'}{\beta} + r_e\right)$$

Effect of C_E at gain



$$A_V = Gain = \frac{-\beta(R_C \| r_o)}{(\beta+1)(r_e + R_E)} \cong \frac{-R_C}{r_e + R_E}$$

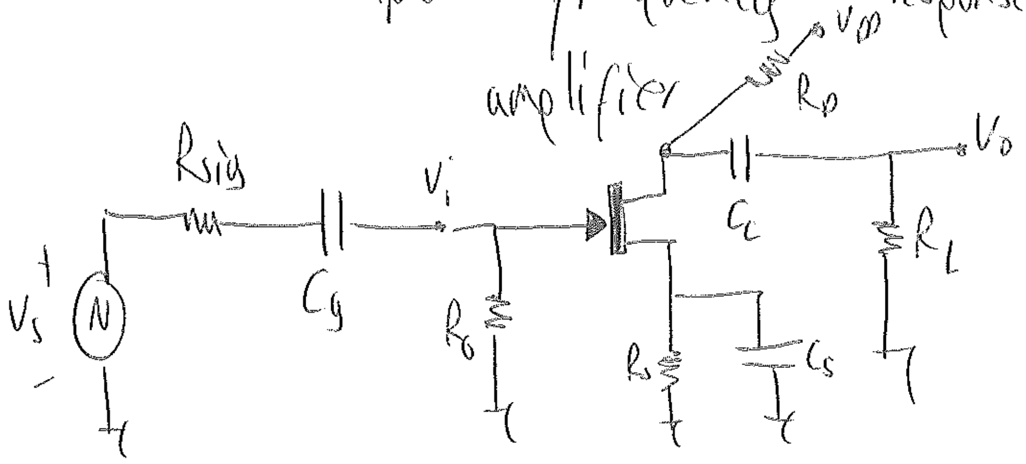
if $R_E = 0$ $A_V = Gain \cong \frac{-R_C}{r_e}$ (nearly maximum gain

*At low frequecies bypass capacitor C_E is open-circuit and for gain we use $A_G = \frac{-R_C}{r_e + R_E}$ → (minimum gain)

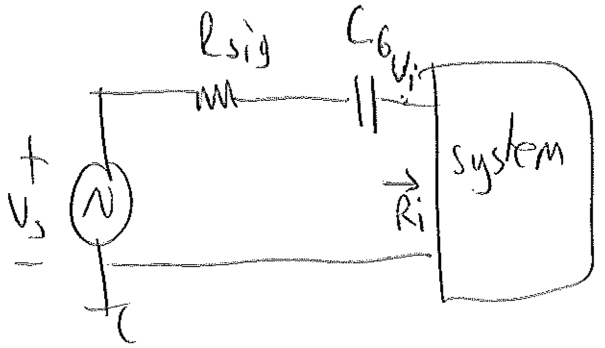
As frequency increases reactance of C_E decreases reducing paralel impedance of R_E and C_E until the resistor R_E is effectively shorted out by C_E that results in maximum midband gain $A_V \cong \frac{-R_C}{r_o}$

at f_{L_E} the gain will be 3dB below the midband gain with R_E "shorted out"

Low frequency response of FET amplifier



For C_g circuit to determine effect of C_G on the low frequency response



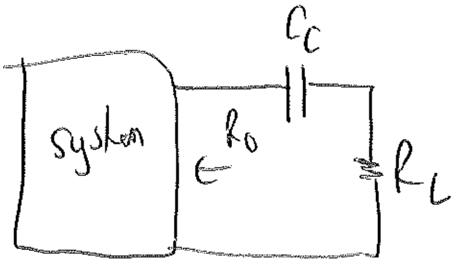
$$f_{LG} = \frac{1}{2\pi (R_{sig} + R_i) C_g}$$

$$R_i = R_G \left(\begin{array}{c} \text{due to} \\ \text{ac analysis} \end{array} \right)$$

generally $R_G >> R_{sig}$

since R_i is large f_{LG} is small

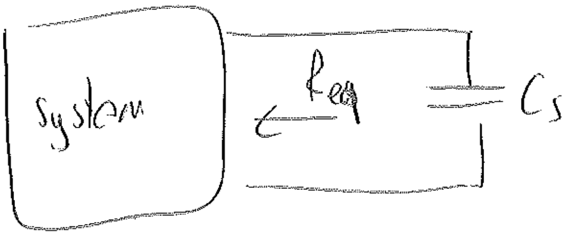
For C_c : circuit to determine effect of C_c on low-frequency response



$$f_{Lc} = \frac{1}{2\pi (R_o + R_L) C_c}$$

$$R_o = R_D \,||\, r_d$$

C_s: Effect of C_s on the low frequency response



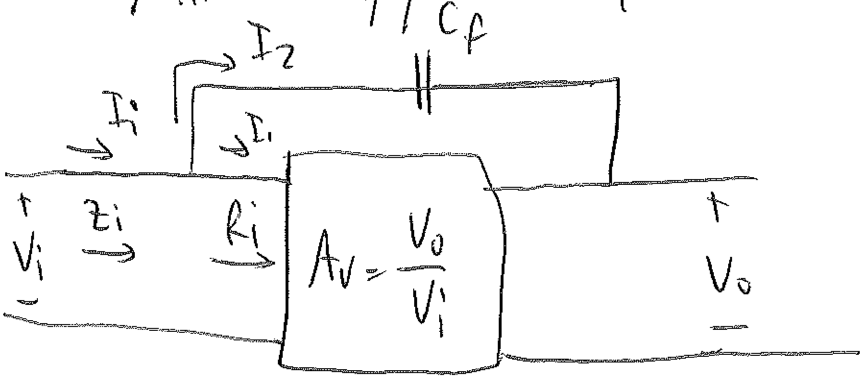
$$f_{L_S} = \frac{1}{2\pi R_{eq} C_s}$$ → low cut-off frequency

$$R_{eq} = \frac{R_s}{1 + R_s(1+g_m r_d)/(r_d + R_D \| R_L)}$$

if $r_d \to \infty$ $\quad R_{eq} = R_s \| \frac{1}{g_m}$

Check examples 9.11 for low frequency response of BJT and 9.12 for low frequency response of FET

Miller effect capacitance



$I_i = I_1 + I_2 \quad I_i = \frac{V_i}{Z_i} \quad I_1 = \frac{V_i}{R_i}$

$$I_2 = \frac{V_i - V_o}{X_{c_f}} = \frac{V_i - A_v V_i}{X_{c_f}} = \frac{(1 - A_v) V_i}{X_{c_f}}$$

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{(1-A_v)V_i}{X_{cf}} \qquad X_{cf} = \frac{1}{\omega C_f}$$

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{(1-A_v)}{X_{cf}}$$

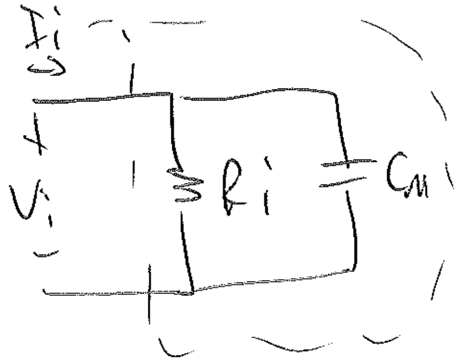
$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{\frac{X_{cf}}{(1-A_v)}}$$

$$\frac{X_{cf}}{1-A_v} = \frac{1}{\omega(1-A_v)C_f} = X_{CM}$$

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{CM}}$$

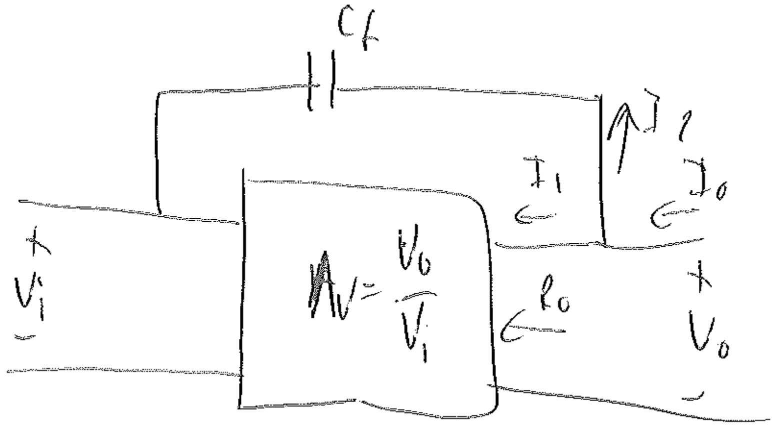
miller effect capacitance $\boxed{C_{M_i} = (1-A_v)C_f}$

new model circuit



$$C_M = (1-A_v)C_f$$

Miller output capacitance



$$I_o = \frac{V_o - \frac{V_o}{A_V}}{X_{cf}} = \frac{V_o\left[1 - \frac{1}{A_V}\right]}{X_{cf}}$$

$$\frac{I_o}{V_o} = \frac{1 - \frac{1}{A_V}}{X_{cf}} \qquad \frac{V_o}{I_o} = \frac{X_{cf}}{1 - \frac{1}{A_V}} = \frac{1}{\omega C_f\left(1 - \frac{1}{A_V}\right)}$$

$$= \frac{1}{\omega C_{Mo}}$$

miller output capacitance

$$C_{Mo} = \left(1 - \frac{1}{A_V}\right) C_f$$

if $A_V \gg 1$ $C_{Mo} \simeq C_f$