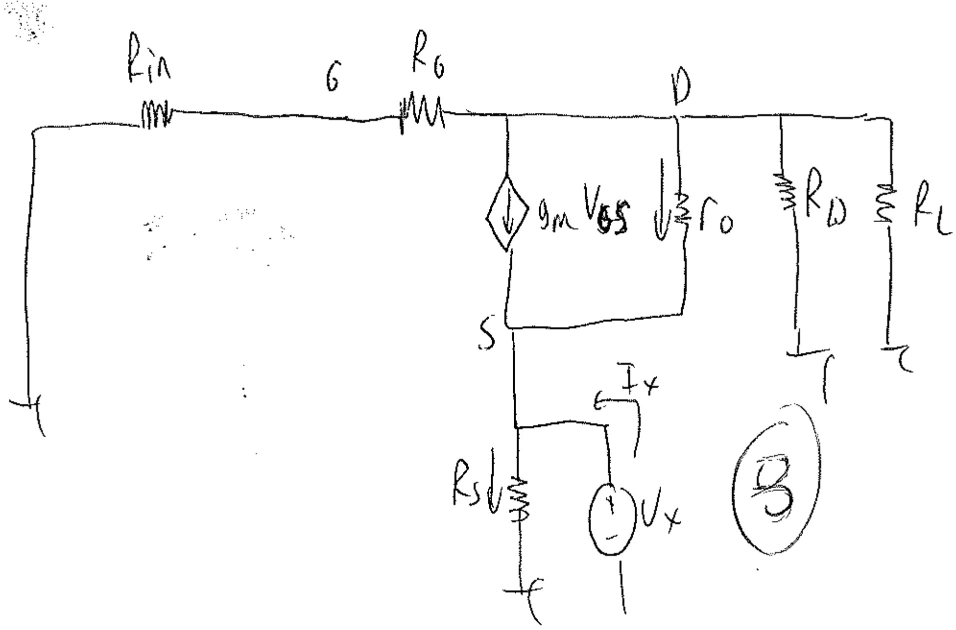
MECE 347 Make-up

2017 – 2018

(Q1) For cut-off frequency due to capacitor C_s short C_g and C_d. Put a test source V_x instead of C_s and calculate current I_x, short V_{in}



(8)

$V_S = V_X$ $\qquad I_x = \frac{V_x}{R_S} - \frac{V_D - V_S}{r_o} - g_m V_{GS}$

<u>At node S</u>

$$I_x = \frac{V_x}{1000} + \frac{V_s}{r_o} - \frac{V_D}{r_o} - g_m(V_G - V_S)$$

$$I_x = \frac{V_x}{1000} + \frac{V_x}{1000} - \frac{V_D}{1000} - \frac{1}{1000}\left(V_G - V_x\right)$$

$$\boxed{I_x = \frac{3}{1000}V_x - \frac{V_D}{1000} - \frac{V_G}{1000}} \ *$$

(10)

At node D

$$\frac{V_D - V_G}{R_G} + g_m V_{GS} + \frac{V_D - V_S}{r_o} + \frac{V_D}{R_D \| R_L} = 0$$

$$\frac{V_D - V_G}{500} + \frac{1}{1000}(V_G - V_S) + \frac{V_D - V_S}{1000} + \frac{V_D}{500} = 0$$

$$V_D\left[\frac{1}{500} + \frac{1}{500} + \frac{1}{1000}\right] + V_G\left[\frac{1}{1000} - \frac{1}{500}\right] - \frac{2}{1000} V_S = 0$$

$$V_D\left[\frac{5}{1000}\right] - V_G \frac{1}{1000} - \frac{2}{1000} V_S = 0$$

$$\boxed{5V_D - V_G + 2V_S = 0} \quad (4) \quad \boxed{5V_D - V_G + 2V_X = 0} \ **$$

At node G

$$\frac{D - G}{R_G} = \frac{G - 0}{R_{in}} \qquad \frac{D - G}{500} = \frac{G - 0}{500}$$

$$\boxed{V_D = 2V_G} \ ***$$

put *** in **

$$5(2V_G) - V_G + 2V_S = 0$$

$$\boxed{9V_G + 2V_S = 0} \quad (4) \quad V_G = \frac{-2V_X}{9}$$

$$I_X = \frac{3}{1000} V_X - \frac{2V_G}{1000} - \frac{V_G}{1000}$$

$$I_X = \frac{3V_X}{1000} - \frac{3V_G}{1000} \qquad I_X = \frac{3V_X}{1000} - \frac{3}{1000}\left(\frac{-2}{9}\right)V_X$$

$$I_X = \left[\frac{3}{1000} + \frac{2}{3000}\right]V_X \quad (2)$$

$$I_x = \left[\frac{9+2}{3000} \right] V_x \qquad \frac{V_x}{I_x} = \frac{3000}{11}\,\Omega$$

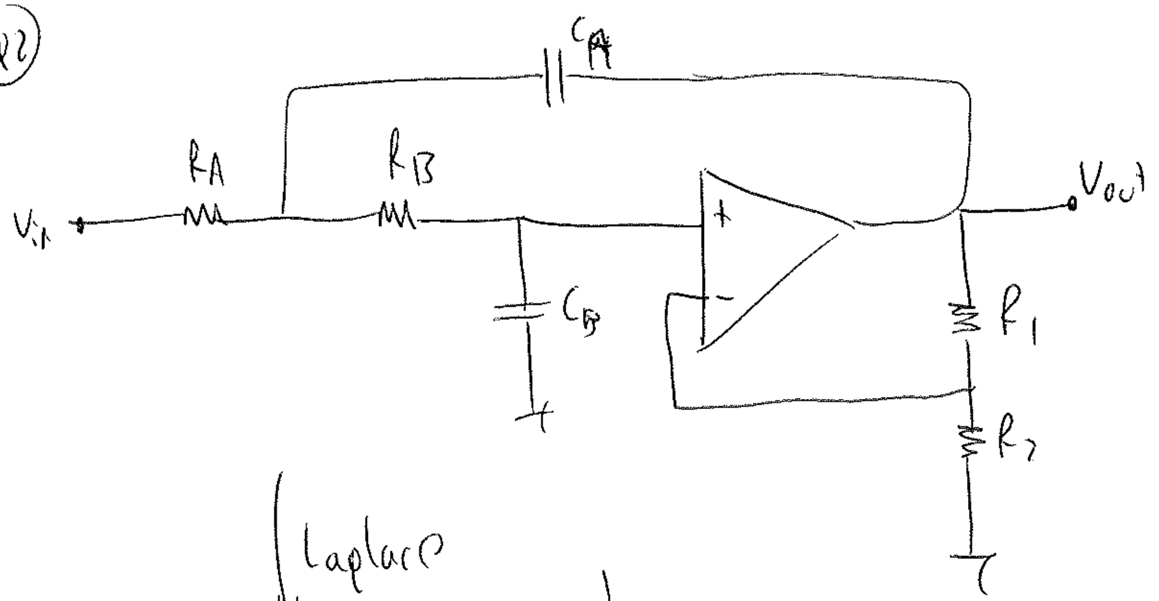
$$R_x = \frac{3000}{11}\,\Omega$$

$$f\text{-cut-off-due-to-}C_s = \frac{1}{2\times\pi\times C_s\times R_x} = \frac{1}{2\times\pi\times 1\times 10^{-6}\times\frac{3000}{11}}$$

$$= \frac{11\times 10^6}{2\times\pi\times 3000} = \frac{11000}{6\times\pi}\ \text{Hertz}$$

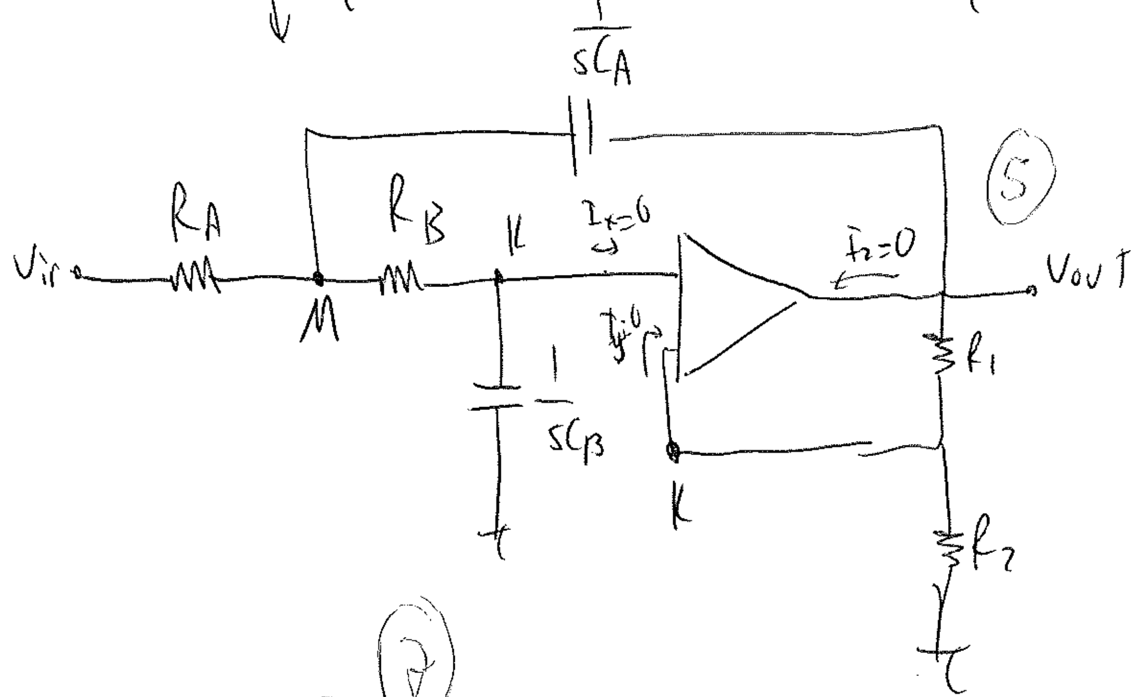
3

Q2



(a)

Laplace



2

$$\boxed{K = V_{out}\frac{R_2}{R_1+R_2}} \quad *$$

$$\frac{M}{R_B + \frac{1}{sC_B}} = \frac{K}{\frac{1}{sC_B}}$$

$$\frac{M sC_B}{sC_B R_B + 1} = K s C_B$$

3

$$\boxed{M = [sC_B R_B + 1] K}$$

**

$$\boxed{\frac{V_{in} - M}{R_A} = \frac{M - K}{R_B} + \frac{M - V_{out}}{\frac{1}{sC_A}}}$$

3

$$(V_{in} - M) R_B = (M - K) R_A + sC_A R_A R_B (M - V_{out})$$

$$\boxed{V_{in} R_B + V_{out}(sC_A R_A R_B) = M[R_A + R_B + sC_A R_A R_B] - K R_A}$$

***

$$V_{in} R_B + V_{out}\left(s C_A R_A R_B\right) = \left(s C_B R_B + 1\right)\left[R_A + R_B + s C_A R_A R_B\right] K - K R_A$$

$$V_{in} R_B + V_{out}\left(s C_A R_A R_B\right) = \left(s^2 C_A C_B R_A R_B^2 + s\left(C_B R_B R_A + C_B R_B R_B + C_A R_A R_B\right) + R_B\right) K$$

$$V_{in} + V_{out}\left(s C_A R_A\right) = \left(s^2 C_A C_B R_A R_B + s\left(C_B R_A + C_B R_B + C_A R_A\right) + 1\right) K$$

$$V_{in} = \left(\frac{R_2}{R_1+R_2}\left[s^2 C_A C_B R_A R_B + s\left(R_A C_B + C_B R_B + C_A R_A\right) + 1\right] - s C_A R_A\right) V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{s^2\left(R_2 C_A C_B R_A R_B\right) + s R_2\left(R_A C_B + R_B C_B + R_A C_A\right) - \left(R_1 + R_2\right) s C_A R_A + R_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{R_2} \cdot \frac{1}{s^2\left(C_A C_B R_A R_B\right) + s\left(R_A C_B + R_B C_B + R_A C_A - \frac{R_1+R_2}{R_2} C_A R_A\right) + 1}$$

(b) $C_A = C_B = C \qquad R_A = R_B = R$ $\qquad$ ~~Let $R_1 = 0$, $R_2 = \infty$~~

$$\frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{R_2} \cdot \frac{1}{s^2\left(C^2 R^2\right) + s\left(3RC - \frac{R_1+R_2}{R_2} RC\right) + 1}$$

Let $R_1 = 0 \quad R_2 = \infty$

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2 C^2 R^2 + s\left(2CR\right) + 1}$$

(c) $\frac{V_{out}}{V_{in}} = \frac{1}{s^2C^2R^2 + s(2CR) + 1} = H(s) = \frac{1}{[sCR+1]^2}$

$H(j\omega) = \frac{1}{[1 - C^2R^2\omega^2] + j\omega(2CR)}$ 1 $\quad |H(j\omega)| = \frac{1}{[(1-C^2R^2\omega^2)^2 + (\omega 2CR)^2]^{\frac{1}{2}}}$ 3

$$|H(j\omega)|_{dB} = 20\log 1 - 20\log\left[(1-C^2R^2\omega^2)^2 + (\omega 2CR)^2\right]^{\frac{1}{2}}$$

$$= -20\log\left[(1-\omega^2R^2C^2)^2 + (2RC\omega)^2\right]^{\frac{1}{2}}$$

<u>Approximation</u>

$0 < \omega \leq \frac{1}{CR}$

$|H(j\omega)|_{dB} \simeq -20\log 1 = 0\,dB$

$\frac{1}{CR} < \omega$

$$|H(j\omega)|_{dB} \simeq -20\log(\omega^2R^2C^2) = -20\log\omega^2 - 20\log R^2C^2$$

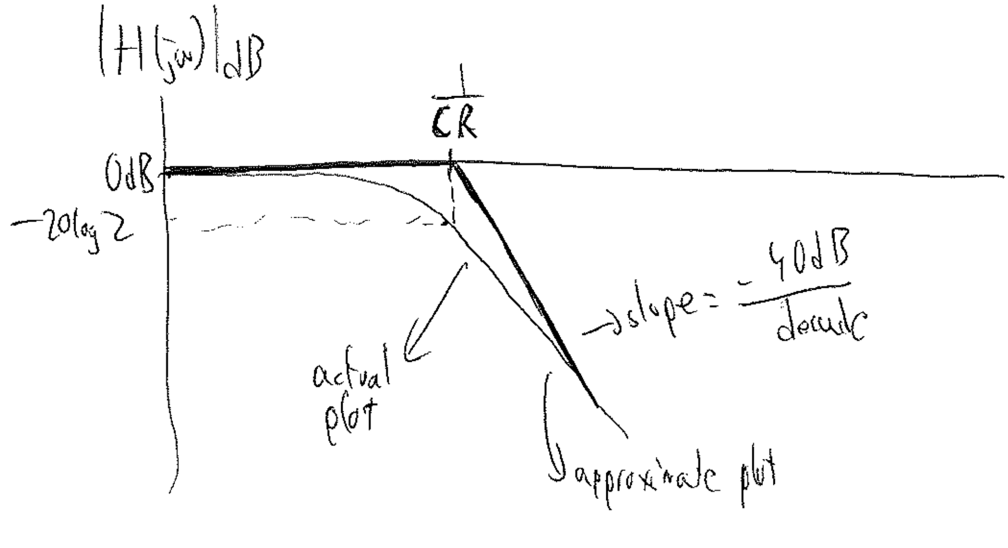
$$= -40\log\omega - 20\log R^2C^2$$

<u>Real Value</u>

let $\omega = RC$

$$|H(j\omega)|_{dB} = -20\log\left[\left(1-\frac{1}{R^2C^2}R^2C^2\right)^2 + \left(2RC\frac{1}{RC}\right)^2\right]^{\frac{1}{2}}$$

$$= -20\log[0+4]^{\frac{1}{2}} = -20\log 2$$



(d) actual cut off frequency (low pass filter)

$$|H(j\omega)|_{max} = |H(j\omega)|\Big|_{w=0} = 1$$

$$\frac{|H(j\omega)|_{max}}{\sqrt{2}} = |H(j\omega)|\Big|_{w=\text{cut-off}} = \frac{1}{\sqrt{2}}$$

$wCR = x$

$$\frac{1}{\left[(1-C^2R^2w^2)^2 + (w2CR)^2\right]^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$$

$$2 = (1-C^2R^2w^2)^2 + (w2CR)^2$$

$$2 = (1-x)^2 + (2x)^2$$

$$2 = 1 + x^2 - 2x + 4x^2$$

$$5x^2 - 2x - 1 = 0$$

$$x = \frac{2 \mp \sqrt{4+20}}{2 \times 5} = \frac{2 \mp \sqrt{24}}{10} = \frac{2 \mp 2\sqrt{6}}{10} = \frac{1 \mp \sqrt{6}}{5}$$

$x \to \frac{1-\sqrt{6}}{5}$ (not possible as $\frac{1-\sqrt{6}}{5} < 0$ and x should be greater than 0)

$x \to \frac{1+\sqrt{6}}{5}$ (possible)

$wCR = x \qquad wCR = \frac{1+\sqrt{6}}{5} \qquad w = \frac{1+\sqrt{6}}{5CR} = \frac{1+\sqrt{6}}{5}\left(\frac{1}{CR}\right) \frac{\text{rad}}{\text{sec}}$