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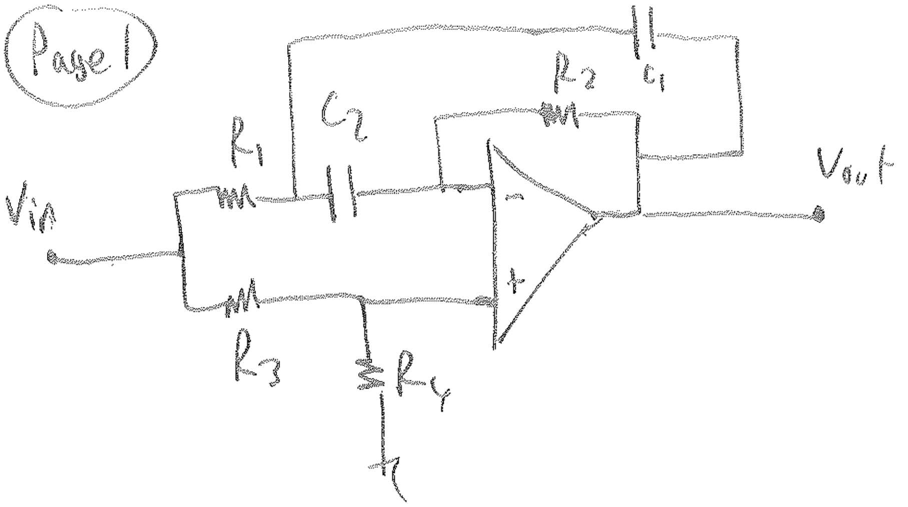
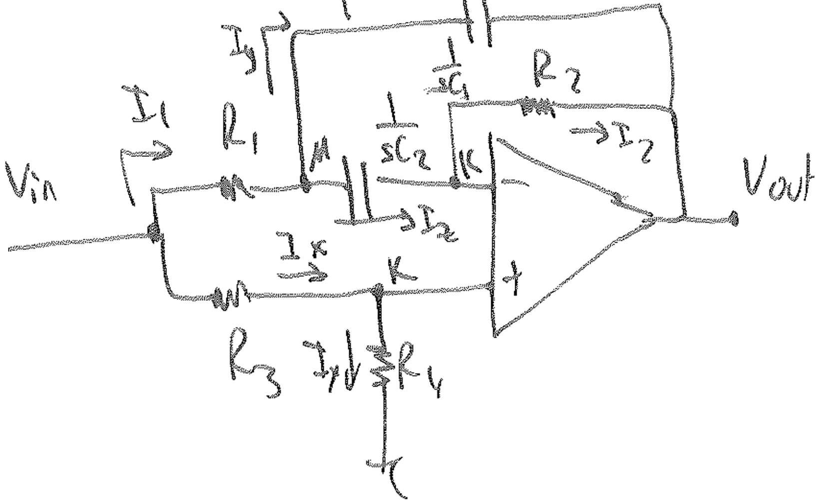


Band-stop filter (multiple feedback)

MECE 397
Week 8 extra notes
Band-stop filter

In Laplace Domain



$$\frac{V_{in}-K}{R_3} = \frac{K}{R_4}$$

Let $R_4 = \frac{R_3}{2}$

$$\boxed{K = V_{in}\frac{R_4}{R_3+R_4}}$$

$$K = \frac{V_{in}}{3}$$

$$\frac{V_{in}-M}{R_1} = \frac{M-K}{\frac{1}{sC_2}} + \frac{M-V_{out}}{\frac{1}{sC_1}}$$

$$\frac{V_{in}}{R_1} = M\left[\frac{1}{R_1}+sC_1+sC_2\right] - KsC_2 - V_{out}\cdot sC_1$$

Let $C_1 = C_2 = C$

$$\boxed{\frac{V_{in}}{R_1} = M\left[\frac{1}{R_1}+2sC\right] - KsC - V_{out}sC}$$

$$\frac{M-K}{\frac{1}{sC_2}} = \frac{K-V_{out}}{R_2}$$

$$\boxed{MsC = K\left[\frac{1}{R_2}+sC\right] - \frac{V_{out}}{R_2}}$$

$$\boxed{V_{in} = M[1+2sCR_1] - KsCR_1 - V_{out}sCR_1}$$

$$\boxed{MsCR_2 = K[1+sCR_2] - V_{out}}$$

$$\boxed{K = \frac{V_{in}}{3}}$$

$$M = \frac{V_{in}}{1+2sCR_1} + \frac{KsCR_1}{1+2sCR_1} + \frac{V_{out}sCR_1}{1+2sCR_1} = \frac{K[1+sCR_2]}{sCR_2} - \frac{V_{out}}{sCR_2}$$

$$V_{in}sCR_2 + Ks^2C^2R_1R_2 + V_{out}s^2C^2R_1R_2 = K[1+sCR_2][1+2sCR_1] - V_{out}(1+2sCR_1)$$

Let $R_1 = R_2 = R$

$$V_{in}sCR + Ks^2C^2R^2 + V_{out}s^2C^2R^2 = K[1+sCR][1+2sCR] - V_{out}(1+2sCR)$$

$$V_{in}sCR + V_{out}[1+2sCR + s^2C^2R^2] = K(1+2s^2C^2R^2 - s^2C^2R^2 + 3sCR)$$

$$V_{in}sCR + V_{out}[1+2sCR+s^2C^2R^2] = K[1+s^2C^2R^2+3sCR]$$

$$V_{in}sCR + V_{out}(1+2sCR+s^2C^2R^2) = \frac{V_{in}}{3}[1+s^2C^2R^2+3sCR]$$

$$V_{in}sCR + V_{out}[1+2sCR+s^2C^2R^2] = V_{in}\left(\frac{1}{3} + \frac{1}{3}s^2C^2R^2 + sCR\right)$$

$$V_{out}(1+2sCR+s^2C^2R^2) = \frac{1}{3}V_{in}[1+s^2C^2R^2]$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{3}\,\frac{1+s^2C^2R^2}{1+2sCR+s^2C^2R^2}$$

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$$\frac{V_{out}}{V_{in}} = \frac{1}{3}\,\frac{1+s^2C^2R^2}{1+2sCR+s^2C^2R^2} = \frac{1}{3}\,\frac{s^2+\frac{1}{C^2R^2}}{s^2+s\frac{2}{CR}+\frac{1}{C^2R^2}}$$

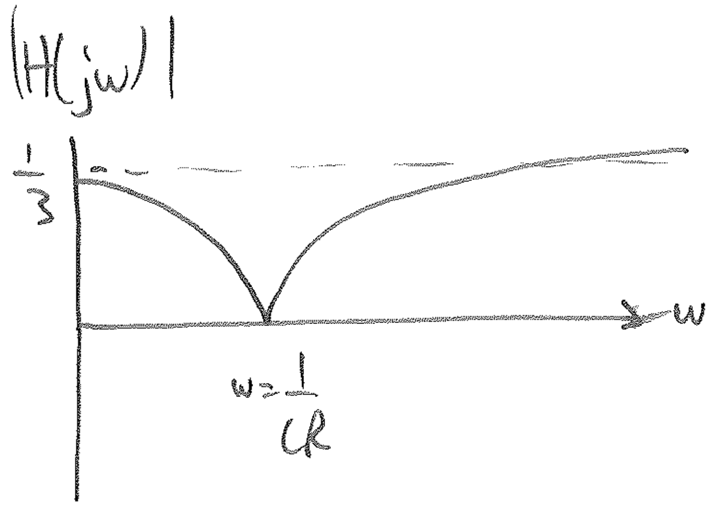
$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{3}\,\frac{\left[-\omega^2+\frac{1}{C^2R^2}\right]}{\left[-\omega^2+\frac{1}{C^2R^2}\right]+j\omega\frac{2}{CR}}$$

if $\omega = 0$ $\quad |H(j\omega)| = \frac{1}{3}$ //

if $\omega \to \infty$ $\quad |H(j\omega)| = \frac{1}{3}$

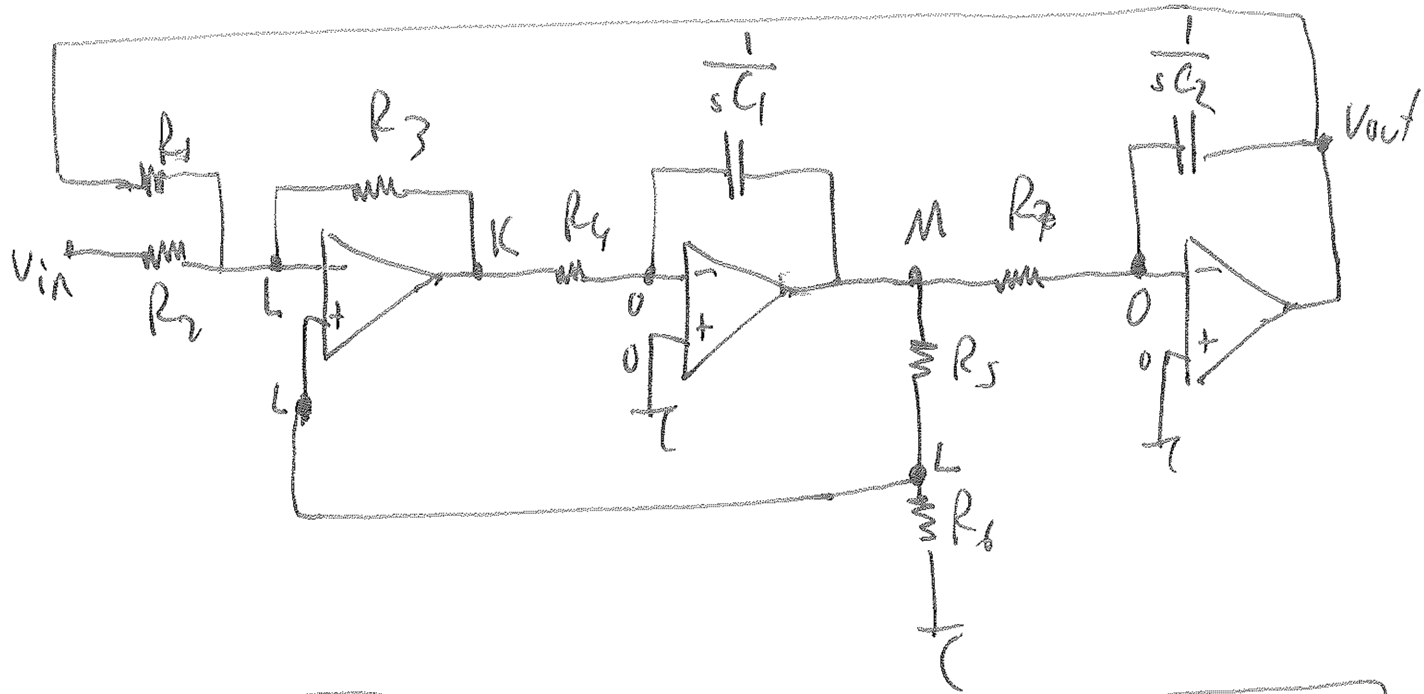
if $\omega = \frac{1}{CR}$ $\quad |H(j\omega)| = 0$

$$|H(j\omega)| = \frac{\left|\frac{1}{C^2R^2}-\omega^2\right|}{\sqrt{\left(\frac{1}{C^2R^2}-\omega^2\right)^2+\left(\frac{2\omega}{CR}\right)^2}}$$



State variable filter

In Laplace domain



$$\boxed{L = M\frac{R_6}{R_5+R_6}}$$

$$\frac{K-0}{R_4} = \frac{0-M}{\frac{1}{sC_1}}$$

$$\boxed{K = -M\left[R_4 sC_1\right]}$$

$$\boxed{K = -M sC_1 R_4}$$

$$\frac{M-0}{R_7} = \frac{0-V_{out}}{\frac{1}{sC_2}}$$

$$\boxed{M = -V_{out}\, sC_2 R_7}$$

$$\frac{V_{in}-L}{R_2} + \frac{V_{out}-L}{R_1} = \frac{L-K}{R_3}$$

$$\frac{V_{in}}{R_2} + \frac{V_{out}}{R_1} = L\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right] - \frac{K}{R_3}$$

$$\frac{V_{in}}{R_2} + \frac{V_{out}}{R_1} = M\frac{R_6}{R_5+R_6}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) + \frac{M sC_1 R_4}{R_3}$$

$R_1 = R_2 = R_3 = R_4 = R_7 \qquad C_1 = C_2 \qquad$ let $\dfrac{R_6}{R_5+R_6} = X$

$$\frac{V_{in}}{R} + \frac{V_{out}}{R} = M \cancel{X}\left[\frac{3}{R}\right] + MsC \quad , \quad \boxed{M = -V_{out}\, sCR}$$

$$\boxed{K = -M\, sCR}$$

$$\boxed{\frac{V_{in}}{R} + \frac{V_{out}}{R} = M\left[\frac{3X}{R} + sC\right]}$$

$$\frac{V_{in}}{R} + \frac{V_{out}}{R} = -V_{out}\, sCR\left(\frac{3X}{R} + sC\right]$$

$$\frac{V_{in}}{R} = -V_{out}\left(\frac{1}{R} + 3sC\cancel{X} + s^2C^2R\right]$$

$$\frac{V_{in}}{R} = -V_{out}\left(\frac{1 + 3sCRX + s^2C^2R^2}{R}\right)$$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{1}{s^2C^2R^2 + 3sCRX + 1}}$$

Low pass filter

put $V_{out} = -\dfrac{M}{sCR}$

Vout

$$\frac{-\dfrac{M}{sCR}}{V_{in}} = -\frac{1}{s^2C^2R^2 + 3sCRX + 1}$$

$$\boxed{\frac{M}{V_{in}} = \frac{sCR}{s^2C^2R^2 + 3sCRX + 1}}$$

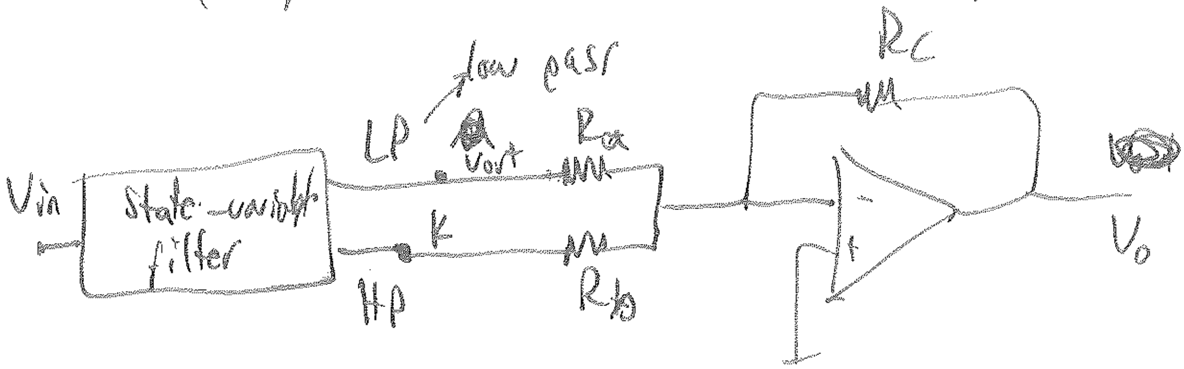
$\rightarrow$ Band-pass filter

$$M = \frac{-K}{sCR}$$

$$\frac{-\frac{K}{sCR}}{V_{in}} = \frac{sCR}{s^2C^2R^2+3sCRX+1}$$

$$\frac{K}{V_{in}} = \frac{-s^2C^2R^2}{s^2C^2R^2+3sCRX+1} \rightarrow \text{High pass filter}$$

Band stop filter (using state variable filter)



$$\frac{V_{out} - 0}{R_a} + \frac{K - 0}{R_b} = \frac{0 - V_o}{R_c}$$

$$V_o = -\frac{V_{out} R_c}{R_a} - K\frac{R_c}{R_b} \qquad V_o = -\frac{R_c}{R_a} V_{out} - \frac{R_c}{R_b} K$$

$$V_o = -\frac{R_c}{R_a}\overbrace{\left[-\frac{V_{in}}{s^2C^2R^2+3sCRK+1}\right]}^{V_{out}} - \frac{R_c}{R_b}\overbrace{\left[\frac{-s^2C^2R^2}{s^2C^2R^2+3sCRK+1}\right]}^{K}$$

$$V_o = \frac{R_c}{R_a}\frac{V_{in}}{s^2C^2R^2+3sCRK+1} + \frac{R_c}{R_b}\left[\frac{s^2C^2R^2\ V_{in}}{s^2C^2R^2+3sCRK+1}\right]$$

$$V_o = \frac{R_cR_b + R_cR_a s^2C^2R^2}{(R_aR_b)\left[s^2C^2R^2+3sCRK+1\right]} V_{in}$$

$$V_o = \frac{R_c R_a C^2 R^2 \left[s^2 + \dfrac{R_c R_b}{R_c R_a C^2 R^2} \right]}{\left(\dfrac{R_a R_b}{\text{[illegible]}} \right) C^2 R^2 \left[s^2 + \dfrac{3sX}{CR} + \dfrac{1}{C^2 R^2} \right]} V_{in}$$

$$V_o = \frac{R_c}{R_b} \left(\frac{s^2 + \dfrac{R_b}{R_a C^2 R^2}}{s^2 + \dfrac{3sX}{CR} + \dfrac{1}{C^2 R^2}} \right) V_{in}$$

$$H(s) = \frac{V_o}{V_{in}} = \frac{R_c}{R_b} \left(\frac{s^2 + \dfrac{R_b}{R_a C^2 R^2}}{s^2 + \dfrac{3sX}{CR} + \dfrac{1}{C^2 R^2}} \right)$$

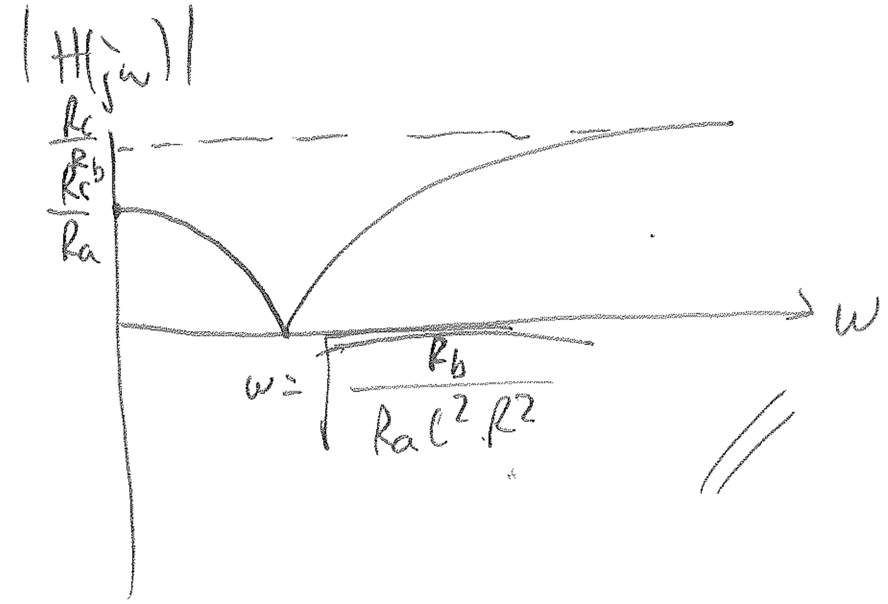
$$H(j\omega) = \frac{R_c}{R_b} \frac{\left[\dfrac{R_b}{R_a C^2 R^2} - \omega^2 \right]}{\left[\dfrac{1}{C^2 R^2} - \omega^2 \right] + \dfrac{3X j\omega}{CR}}$$

$$|H(j\omega)| = \frac{R_c}{R_b} \frac{\left| \dfrac{R_b}{R_a C^2 R^2} - \omega^2 \right|}{\sqrt{\left(\dfrac{1}{C^2 R^2} - \omega^2 \right)^2 + \left(\dfrac{3X\omega}{CR} \right)^2}}$$

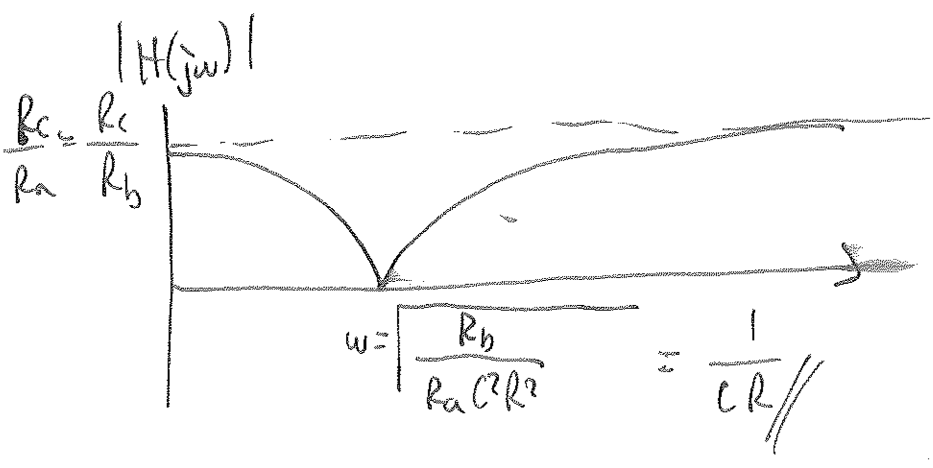
$$|H(j\omega)|\Big|_{\omega=0} = \frac{R_c}{R_b}\,\frac{\frac{R_b}{R_a C^2 R^2}}{\frac{1}{C^2R^2}} = \frac{R_c}{R_a}$$

$$\lim_{\omega\to\infty} |H(j\omega)| = \frac{R_c}{R_b}$$

$$\text{if} \quad \omega = \sqrt{\frac{R_b}{R_a C^2 R^2}} \Rightarrow |H(j\omega)| = 0$$



if $R_a = R_b$



Band pass filter

$$\frac{M}{V_{in}} = \frac{sCR}{s^2C^2R^2+3sCRX+1} = \frac{\frac{s}{CR}}{s^2+\frac{3sX}{CR}+\frac{1}{C^2R^2}} = \frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}$$

$$\omega_0 = \frac{1}{CR} \qquad \omega_0^2 = \frac{1}{C^2R^2}$$

$$2\xi\omega_0 = \frac{3X}{CR} \qquad 2\xi\omega_0 = 3X\omega_0$$

$$\xi = \frac{3X}{2} \qquad \xi = \frac{3}{2}\left[\frac{R_6}{R_5+R_6}\right] \rightarrow \text{damping ratio}$$

$$\omega_0 = \frac{1}{CR} \qquad \omega_{c_1,c_2} = \frac{2\xi\omega_0 \pm \sqrt{4\xi^2\omega_0^2-4\omega_0^2}}{2}$$

$$\omega_{c_1,c_2} = \frac{\xi\omega_0 \mp \omega_0\sqrt{\xi^2-1}}{\cancel{2}}$$

$$\omega_{c_1,c_2} = \frac{\frac{3X}{2}\omega_0 \mp \omega_0\sqrt{\frac{9X^2}{4}-1}}{\cancel{2}}$$

$$|\omega_{c_1}-\omega_{c_2}| = BW = \text{bandwidth} = \cancel{\frac{3X}{2}\omega_0} + \omega_0\sqrt{\frac{9X^2}{4}-1}$$

$$BW = \frac{\frac{3X}{2}+\omega_0\sqrt{\frac{9X^2}{4}-1}}{\cancel{2}} - \frac{\frac{3X}{2}-\omega_0\sqrt{\frac{9X^2}{4}-1}}{\cancel{2}}$$

$$BW = 2\omega_0\sqrt{\frac{9X^2}{4}-1} \qquad Q = \frac{\omega_0}{BW} = \frac{\omega_0}{2\omega_0\sqrt{\frac{9X^2}{4}-1}} \approx \frac{2}{2\times 3X} = \frac{1}{3}\left[\frac{1}{\frac{R_6}{R_5+R_6}}\right]$$

$$R = \frac{1}{3}\,\frac{R_5+R_6}{R_6} = \frac{1}{3}\left[\frac{R_5}{R_6}+1\right] = \frac{1}{3}\left[\frac{R_5}{R_6}+1\right]$$