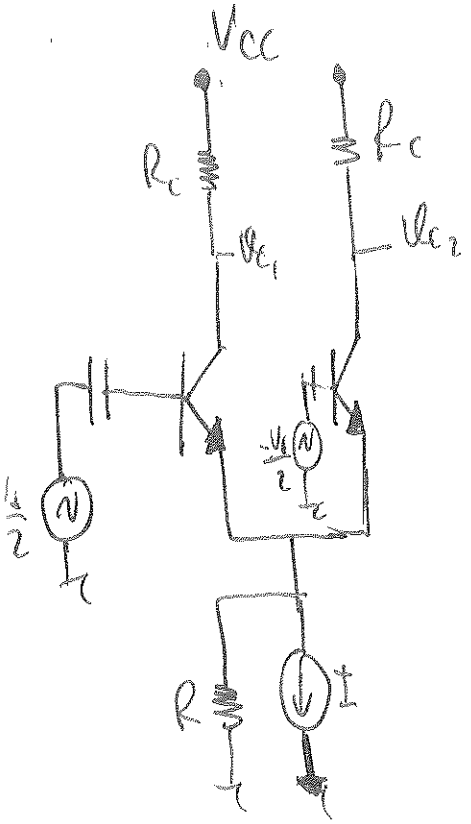
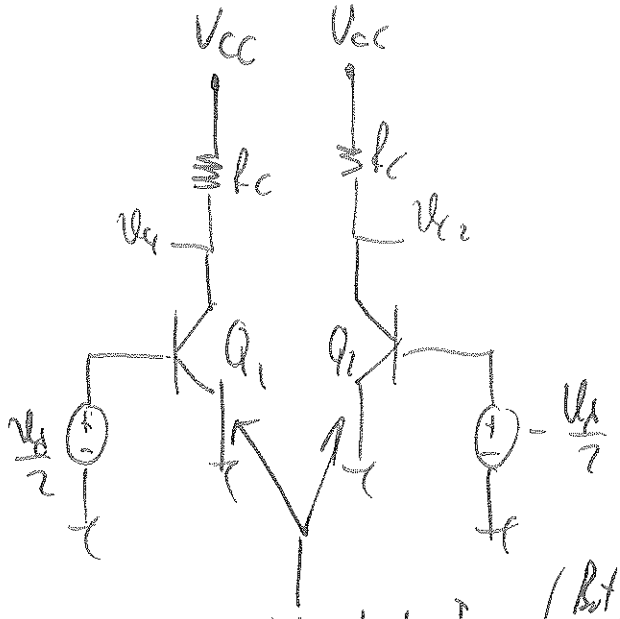
Equivalence of the diff amplifier to a common-emitter amplifier

DC circuit



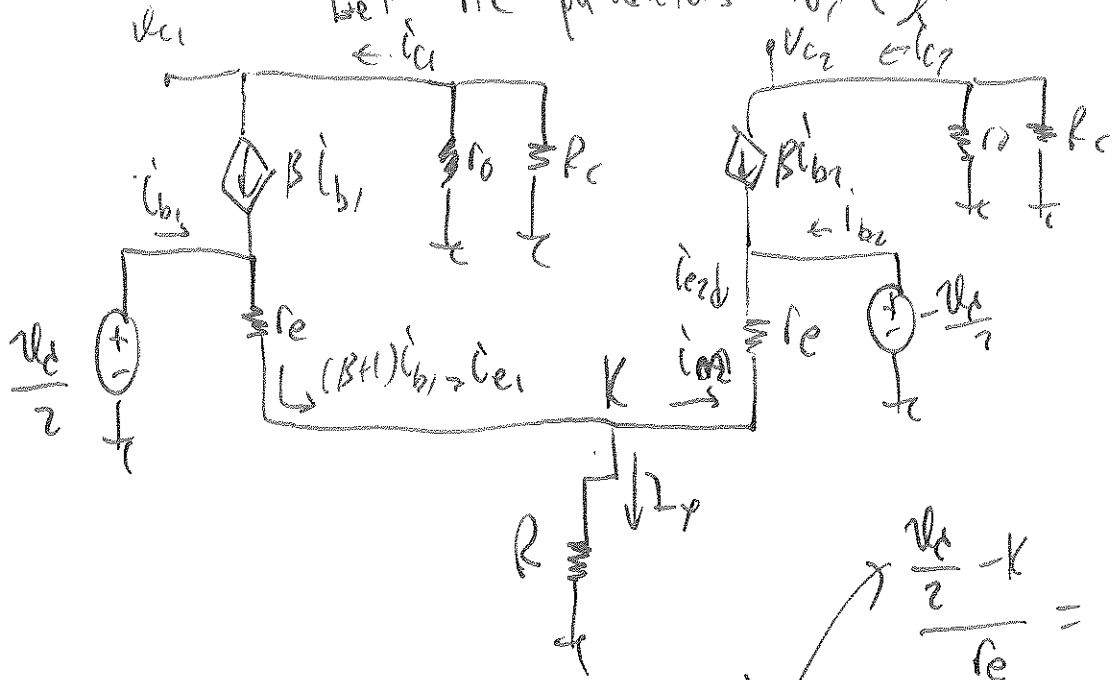
$\equiv$

V_d small signal



(But parts are biased at $\frac{I}{2}$)

but Ac parameters r_o, r_e, β



$$\frac{\frac{V_d}{2} - K}{r_e} = \frac{K - 0}{R} + \frac{K - \left(-\frac{V_d}{2}\right)}{r_e}$$

$K = 0$

$$\frac{\frac{V_d}{2} - K}{r_e} = \frac{K}{R} + \frac{K + \frac{V_d}{2}}{r_e}$$

$$i_{e_1} = (\beta+1)\, i_{b_1} = \frac{\frac{v_d}{2} - K}{r_e} = \frac{v_d}{2r_e} \qquad i_{e_2} = \frac{0 - \left(-\frac{v_d}{2}\right)}{r_e} = +\frac{v_d}{2r_e}$$

$$i_{b_1} = \frac{v_d}{2r_e(\beta+1)} \qquad i_{e_2} = -i_{b_2} = -\frac{v_d}{2r_e}$$

$$i_{b_2} = \frac{i_{e_2}}{\beta+1} = -\frac{v_d}{2r_e(\beta+1)}$$

$$i_{c_1} = \beta i_{b_1} \qquad i_{c_2} = \beta i_{b_2} = -\frac{v_d \beta}{2r_e(\beta+1)}$$

$$v_{c_1} = -[r_o \| R_c]\, i_{c_1} = -[r_o \| R_c]\, \beta i_{b_1} = -[r_o \| R_c]\, \beta \frac{v_d}{2r_e(\beta+1)}$$

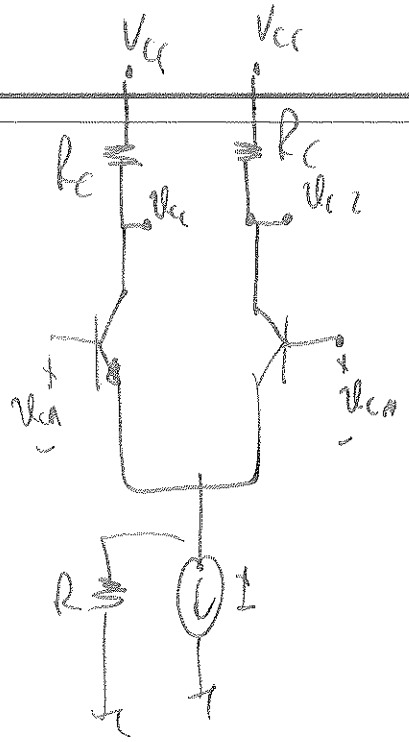
$$v_{c_2} = -[r_o \| R_c]\, i_{c_2} = \left(-[r_o \| R_c]\right)\left(-\frac{v_d \beta}{2r_e(\beta+1)}\right)$$

$$v_{diff}: \quad v_{c_2} - v_{c_1} = -[r_o \| R_c]\, \frac{v_d \beta}{2r_e(\beta+1)} \times 2 =$$

$$\frac{v_{diff}}{v_d} = -\frac{(r_o \| R_c)\ \beta}{r_e(\beta+1)} \simeq -\frac{(R_c \| r_o)}{r_e}$$

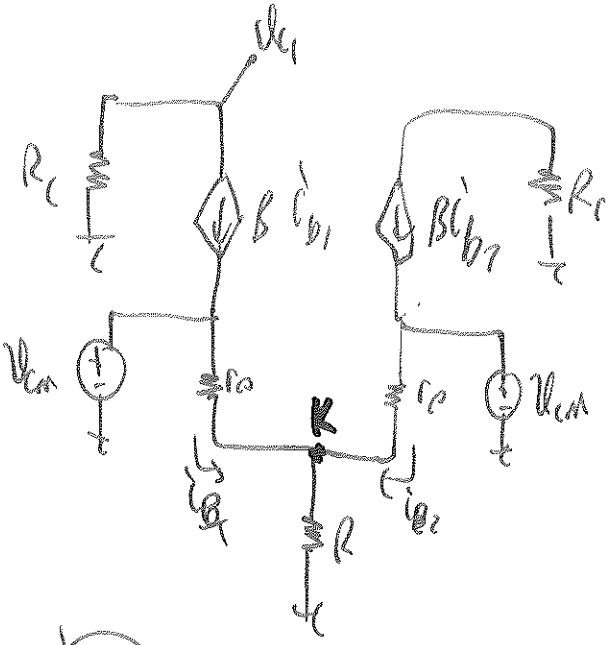
$$\simeq -g_m (R_c \| r_o)$$

Common mode gain



DC circuit

AC circuit



$$v_{c1} = -\beta i_{b1} R_C$$

$$\frac{V_{cm} - K}{r_e} + \frac{V_{cm} - K}{r_e} = \frac{K}{R}$$

$$\frac{2V_{cm}}{r_e} - K\left[\frac{1}{r_e} + \frac{1}{r_e} + \frac{1}{R}\right]$$

$$\frac{2V_{cm}}{r_e} = K\left[\frac{2R + r_e}{r_e R}\right]$$

$$K = \frac{2V_{cm} R}{r_e + 2R} = \frac{2V_{cm} R}{r_e + 2R}$$

$$i_{b1} = \frac{V_{cm} - K}{r_e(\beta+1)} = \frac{V_{cm} - \frac{2V_{cm} R}{r_e + 2R}}{r_e(\beta+1)}$$

$$i_{b1} = \frac{V_{cm} r_e}{r_e(r_e + 2R)(\beta+1)} = \frac{V_{cm}}{(r_e + 2R)(\beta+1)}$$

$$v_{c1} = -\beta i_{b1} R_C \rightarrow v_{c1} = -\frac{V_{cm}(\beta) R_C}{(r_e + 2R)(\beta+1)}$$